DSA 8020 R Session 6: Non-parametric Regression and Shrinkage Methods

Whitney

Contents

Non-parametric Regression: Motorcycle Accident Simulation Data

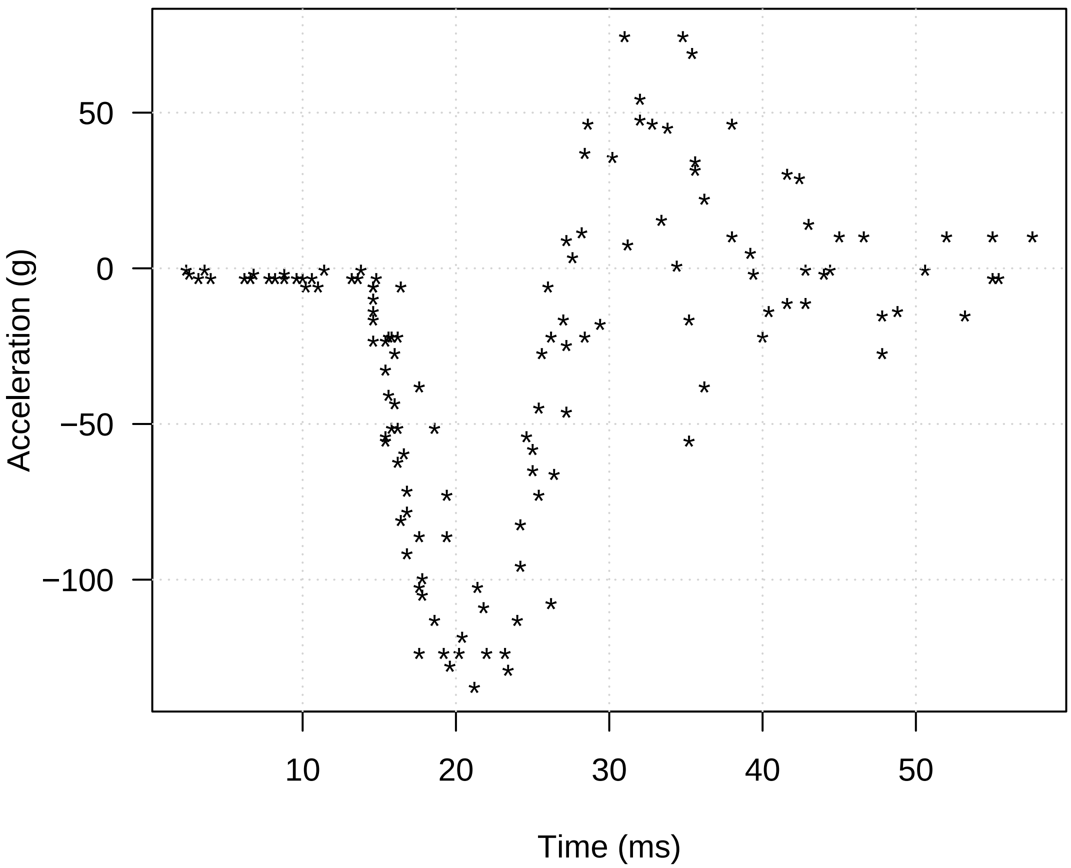
A data frame containing a series of measurements of head acceleration in a simulated motorcycle accident, which is used for testing crash helmets.

- `times`: time in milliseconds after impact
- `accel`: head acceleration in g

Data Source: Silverman, B. W. (1985) Some aspects of the spline smoothing approach to non-parametric curve fitting. *Journal of the Royal Statistical Society series B* 47, 1–52.

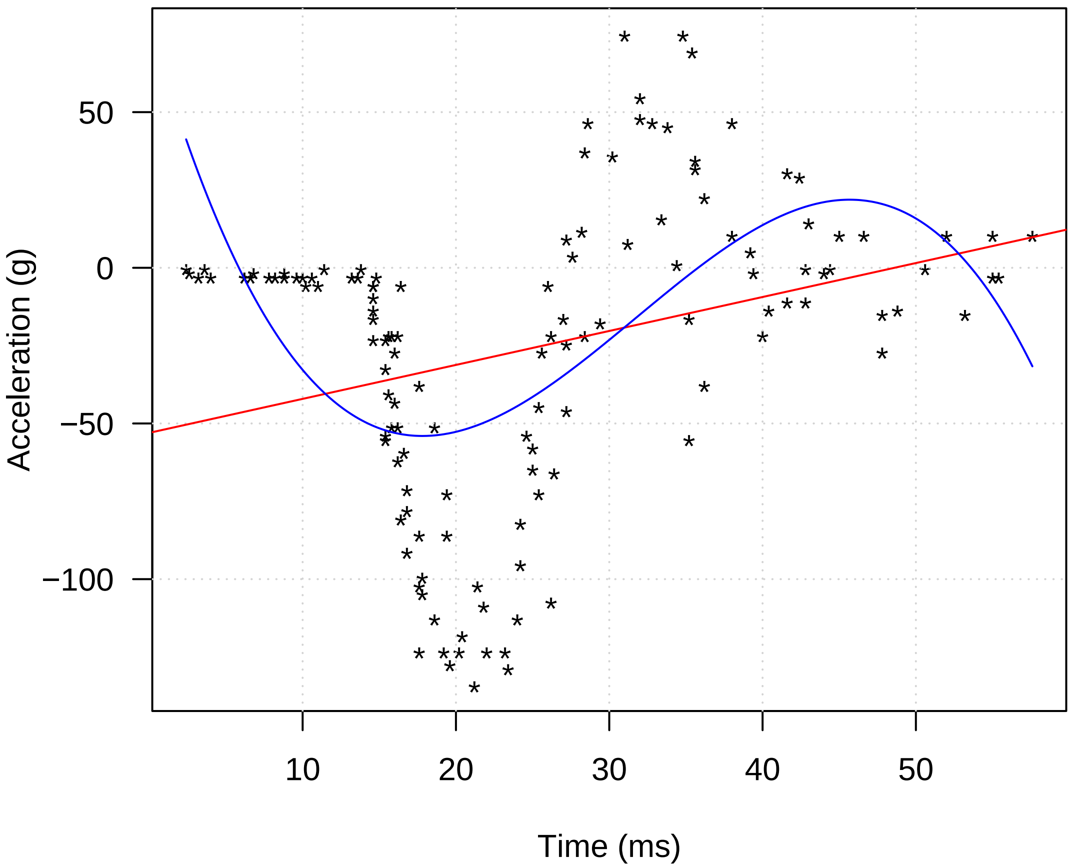
Load and plot the data

```r
library(MASS)
data(mcycle)
attach(mcycle)
plot(times, accel, pch = "*", cex = 1, las = 1,
     xlab = "Time (ms)", ylab = "Acceleration (g)")
grid()
```



Linear and polynomial regression fits

```r
rg <- range(times)
xg = seq(rg[1], rg[2], 0.1) # prediction grids
plot(times, accel, pch = "*", cex = 1, las = 1,
     xlab = "Time (ms)", ylab = "Acceleration (g)")
grid()
lmFit <- lm(accel ~ times, data = mcycle)
abline(lmFit, col = "red")
Cub.polyFit <- lm(accel ~ poly(times, 3), data = mcycle)
Cub.polyPred <- predict(Cub.polyFit, data.frame(times = xg))
lines(xg, Cub.polyPred, col = "blue")
```

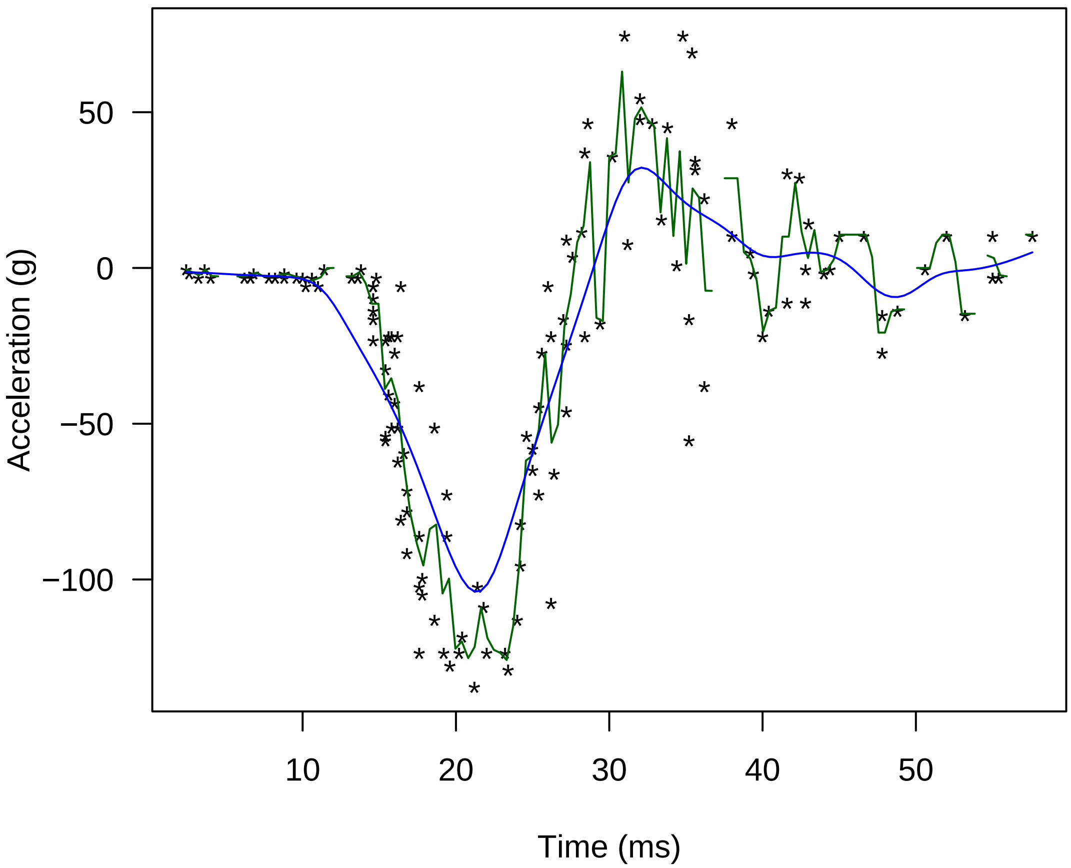


Kernel regression

$\hat{f}(x) = \hat{\mathbb{E}}(y|x) = \frac{\sum_{i=1}^{n} K_h((x-x_i)/h)y_i}{\sum_{i=1}^{n} K_h((x-x_i)/h)}$, where K_h is a kernel with a bandwidth h.

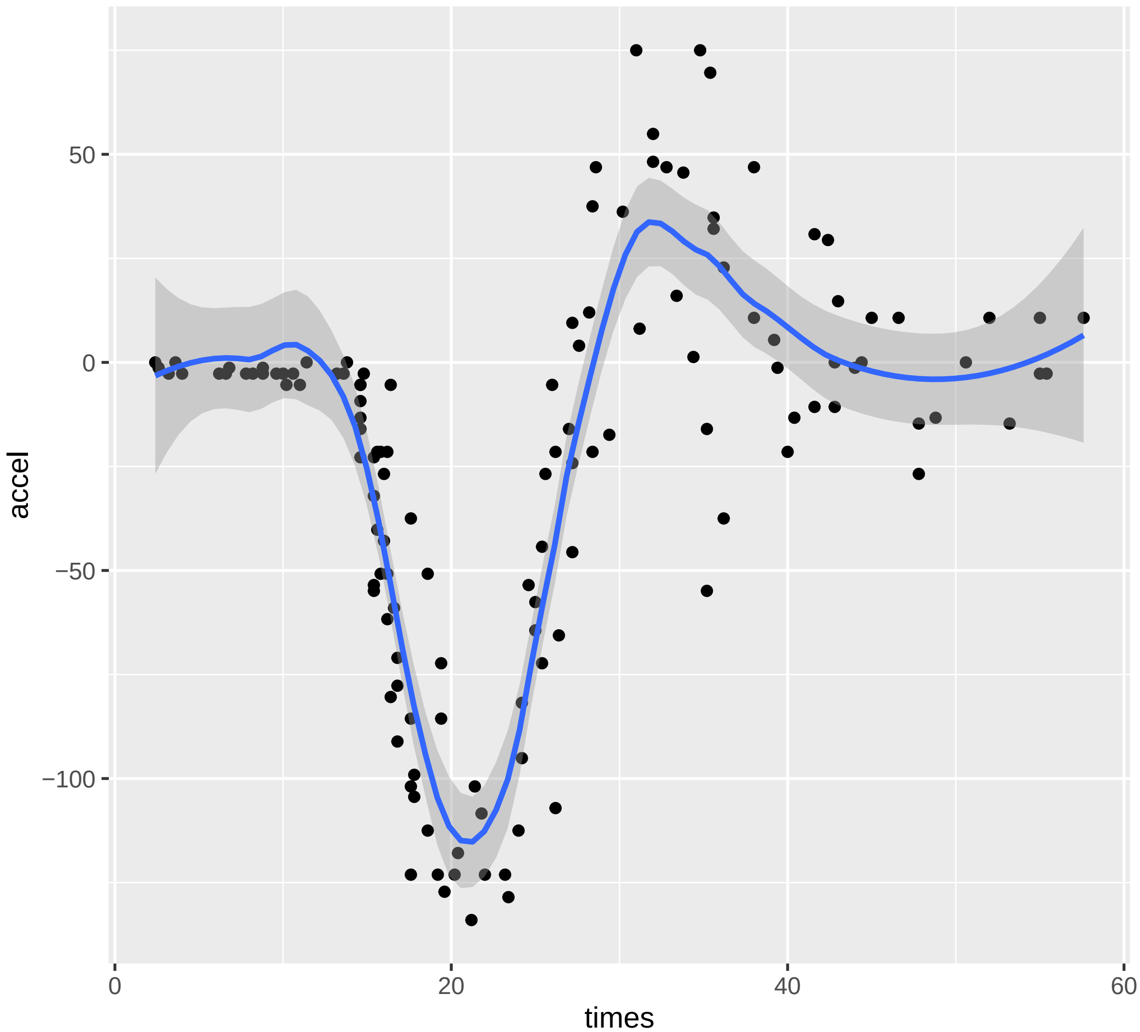
```
KernFit <- with(mcycle, ksmooth(times, accel, kernel = "normal", bandwidth = 0.5))
KernFit2 <- with(mcycle, ksmooth(times, accel, kernel = "normal", bandwidth = 5))

plot(times, accel, pch = "*", cex = 1, las = 1,
     xlab = "Time (ms)", ylab = "Acceleration (g)")
lines(KernFit$x, KernFit$y, col = "darkgreen")
lines(KernFit2$x, KernFit2$y, col = "blue")
```



Local Polynomial Regression Fitting (*loess*)

```
library(ggplot2)
plot <- ggplot(aes(x = times, y = accel), data = mcycle)
plot <- plot + geom_point()
(plot <- plot + geom_smooth(method = "loess", degree = 2, span = 0.4, se = TRUE))
```



Regression Splines

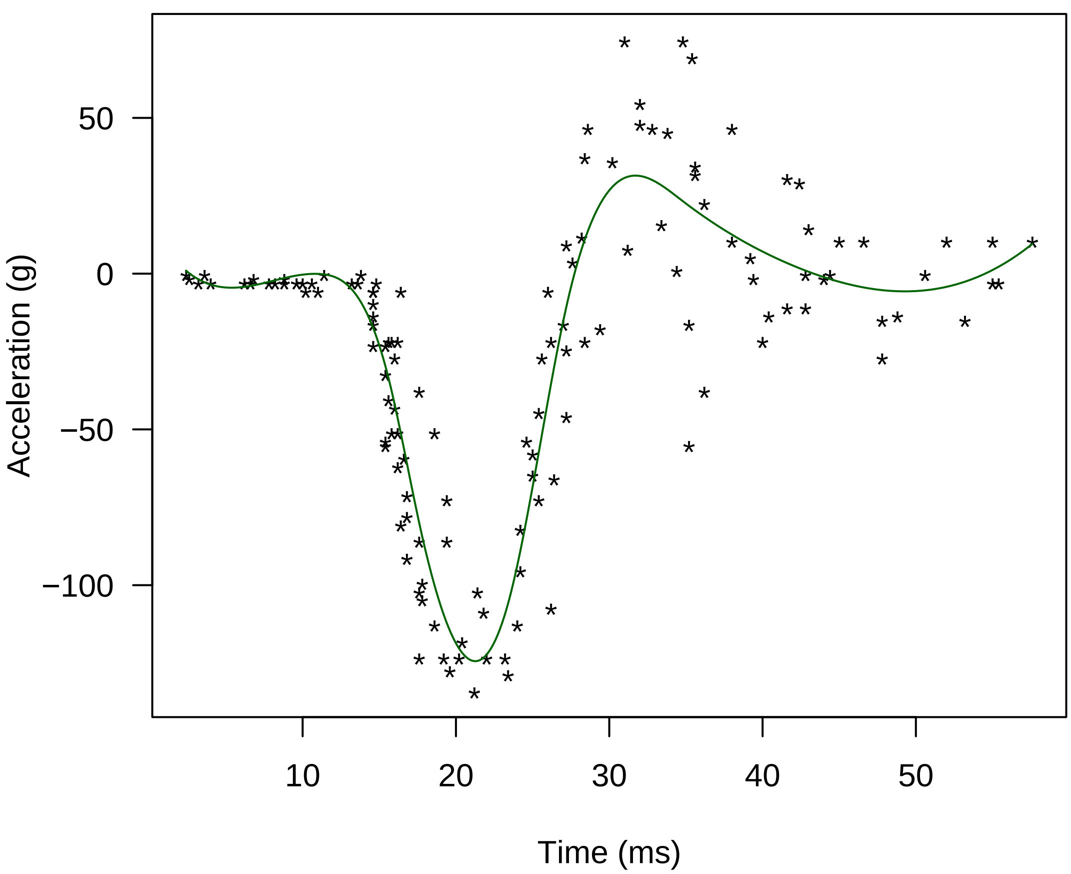
```
library(splines)
RegSplineFit <- lm(accel ~ bs(times, df = 10), data = mcycle)
summary(RegSplineFit)
```

```
##
## Call:
## lm(formula = accel ~ bs(times, df = 10), data = mcycle)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -76.673 -12.362  -0.557  13.139  51.740
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)            0.9312    14.4492   0.064  0.94872
## bs(times, df = 10)1  -12.2008    37.5144  -0.325  0.74556
## bs(times, df = 10)2    6.2223    23.6415   0.263  0.79284
```

```
## bs(times, df = 10)3    -7.3726    18.2652  -0.404  0.68718    
## bs(times, df = 10)4  -118.7497    17.9975  -6.598 1.13e-09 ***
## bs(times, df = 10)5  -152.4486    20.0955  -7.586 7.25e-12 ***
## bs(times, df = 10)6    50.0827    18.7966   2.664  0.00875 ** 
## bs(times, df = 10)7    19.4271    19.3827   1.002  0.31819    
## bs(times, df = 10)8    -8.1814    23.9354  -0.342  0.73308    
## bs(times, df = 10)9   -11.1443    29.2202  -0.381  0.70358    
## bs(times, df = 10)10    8.6378    23.6119   0.366  0.71513    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 22.68 on 122 degrees of freedom
## Multiple R-squared:  0.7964, Adjusted R-squared:  0.7797 
## F-statistic: 47.72 on 10 and 122 DF,  p-value: < 2.2e-16
```

```
RegSplinePred <- predict(RegSplineFit, data.frame(times = xg))

plot(times, accel, pch = "*", cex = 1, las = 1,
     xlab = "Time (ms)", ylab = "Acceleration (g)")
lines(xg, RegSplinePred, col = "darkgreen")
```



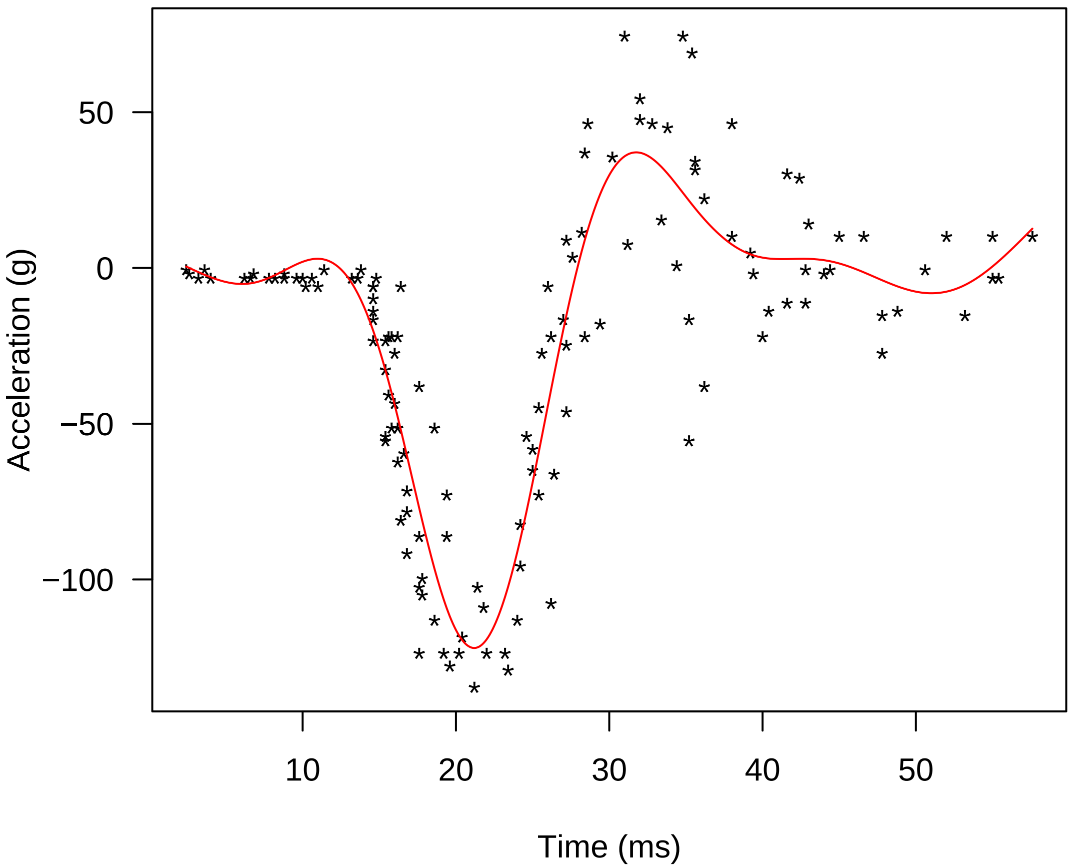
Generalized additive models

```r
library(mgcv)
GAMFit <- gam(accel ~ s(times), data = mcycle)
summary(GAMFit)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## accel ~ s(times)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -25.546      1.951   -13.1   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##            edf Ref.df     F p-value
## s(times) 8.693  8.972 53.52  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.783   Deviance explained = 79.8%
## GCV = 545.78  Scale est. = 506       n = 133
```

```r
GAMpred <- predict(GAMFit, data.frame(times = xg))

plot(times, accel, pch = "*", cex = 1, las = 1,
     xlab = "Time (ms)", ylab = "Acceleration (g)")
lines(xg, GAMpred, col = "red")
```



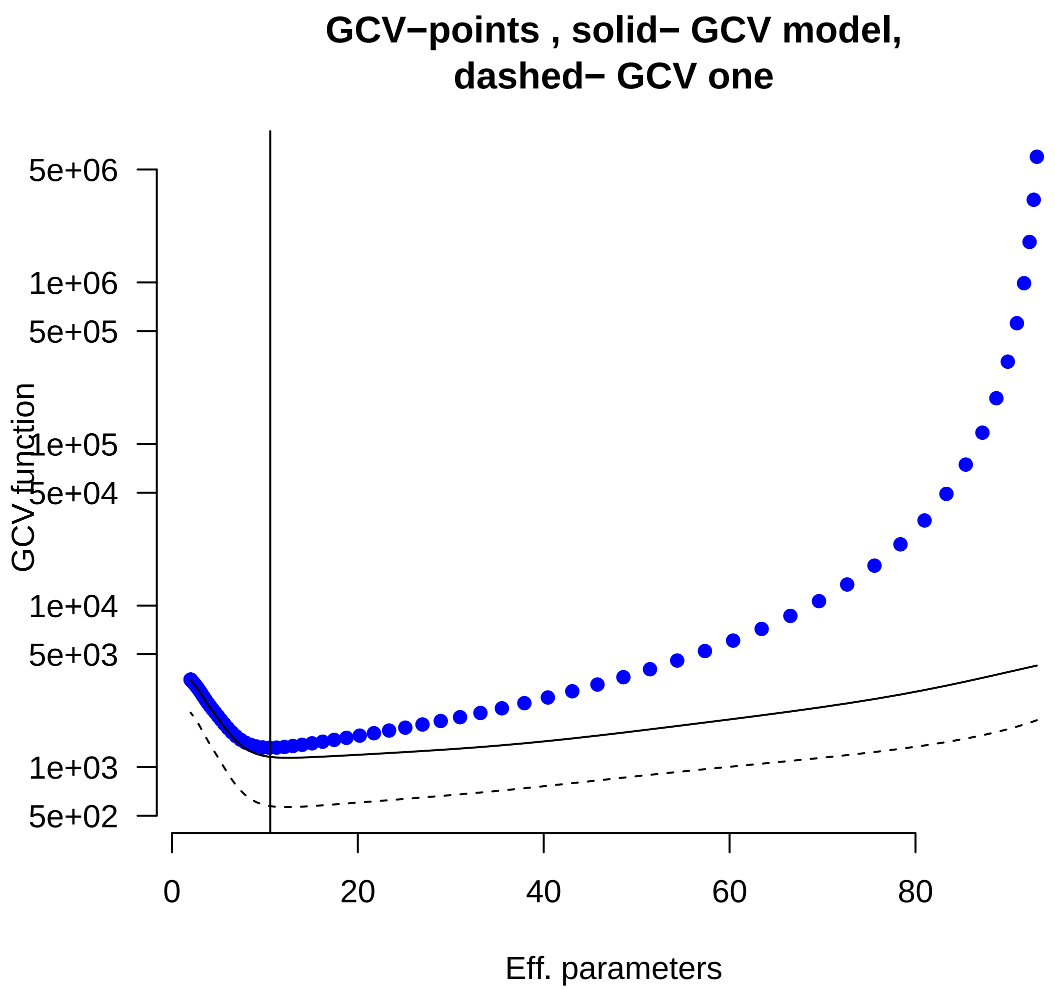
Smoothing splines

```
library(fields)
SpFit <- sreg(times, accel)
summary(SpFit)
```

```
## CALL:
## sreg(x = times, y = accel)
##
##  Number of Observations:             133
##  Number of unique points:            133
##  Eff. degrees of freedom for spline: 10.6
##  Residual degrees of freedom:        122.4
##  GCV est. tau                        22.97
##  Pure error tau                      24.49
##  lambda                              0.3826
##
## RESIDUAL SUMMARY:
##      min    1st Q   median    3rd Q      max
## -78.1500 -13.8800  -0.7238  13.6300  49.6300
##
## DETAILS ON SMOOTHING PARAMETER:
##  Method used:      Cost:
##    lambda       trA       GCV   GCV.one GCV.model    tauHat
```

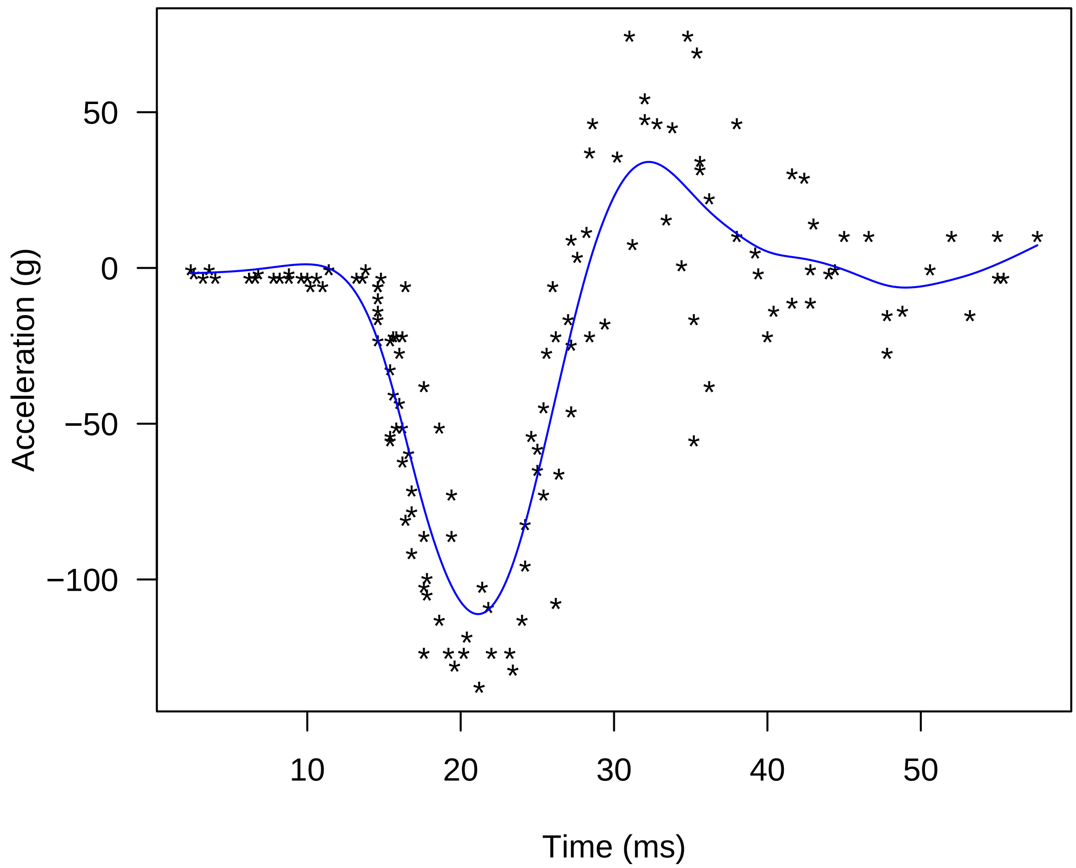
```
##    0.3826   10.5726 1318.0646  573.4152 1156.4850   22.9746
##
##  Summary of estimates for lambda
##            lambda    trA    GCV tauHat converge
## GCV        0.3826 10.573 1318.1  22.97       13
## GCV.model  0.1835 12.467 1142.5  22.64       12
## GCV.one    0.1981 12.253  565.5  22.66       12
## pure error 1.1041  8.375 1380.7  24.49       NA
```

```r
plot(SpFit, which = 3, col = "blue", pch = 16, las = 1)
```



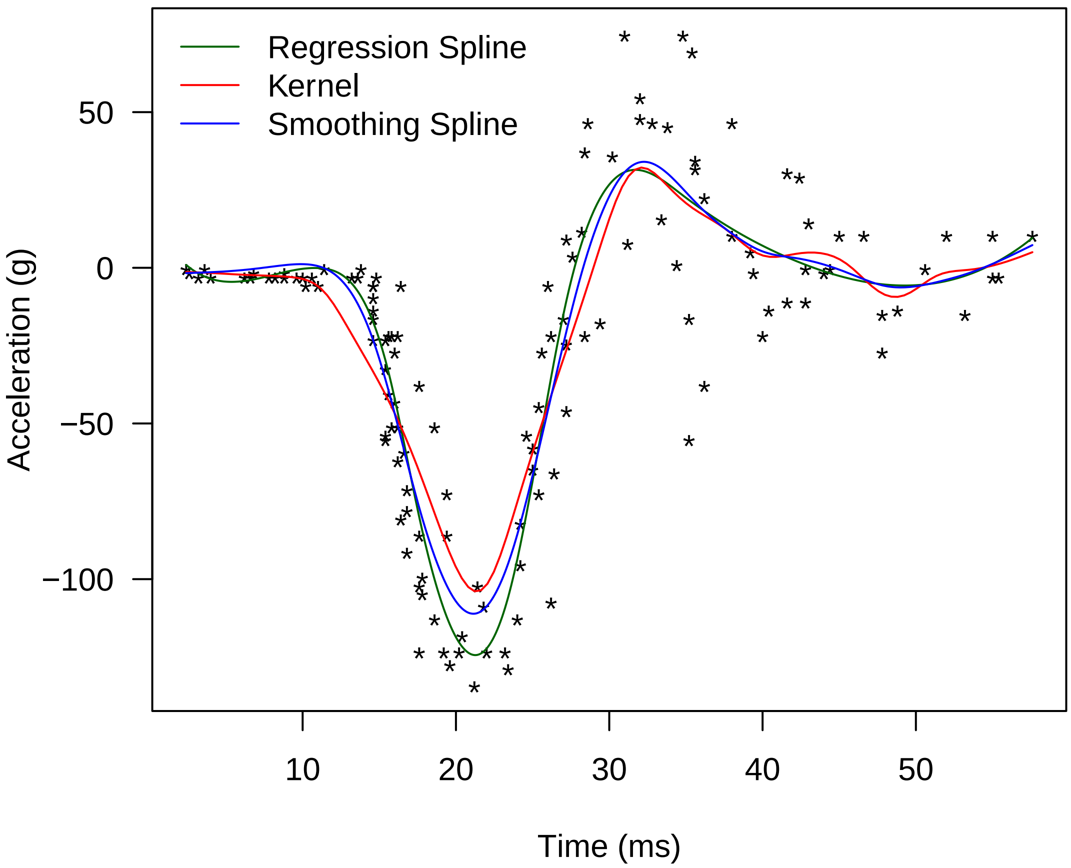

```r
SpPred <- predict(SpFit, xg)

plot(times, accel, pch = "*", cex = 1, las = 1,
     xlab = "Time (ms)", ylab = "Acceleration (g)")
lines(xg, SpPred , col = "blue")
```



Comparing kernel estimator/regression spline/smoothing spline fits

```
plot(times, accel, pch = "*", cex = 1, las = 1,
     xlab = "Time (ms)", ylab = "Acceleration (g)")
lines(xg, RegSplinePred, col = "darkgreen")
lines(KernFit2$x, KernFit2$y, col = "red")
lines(xg, SpPred, col = "blue")
legend("topleft", legend = c("Regression Spline", "Kernel", "Smoothing Spline"),
       col = c("darkgreen", "red", "blue"), lty = 1, bty = "n")
```



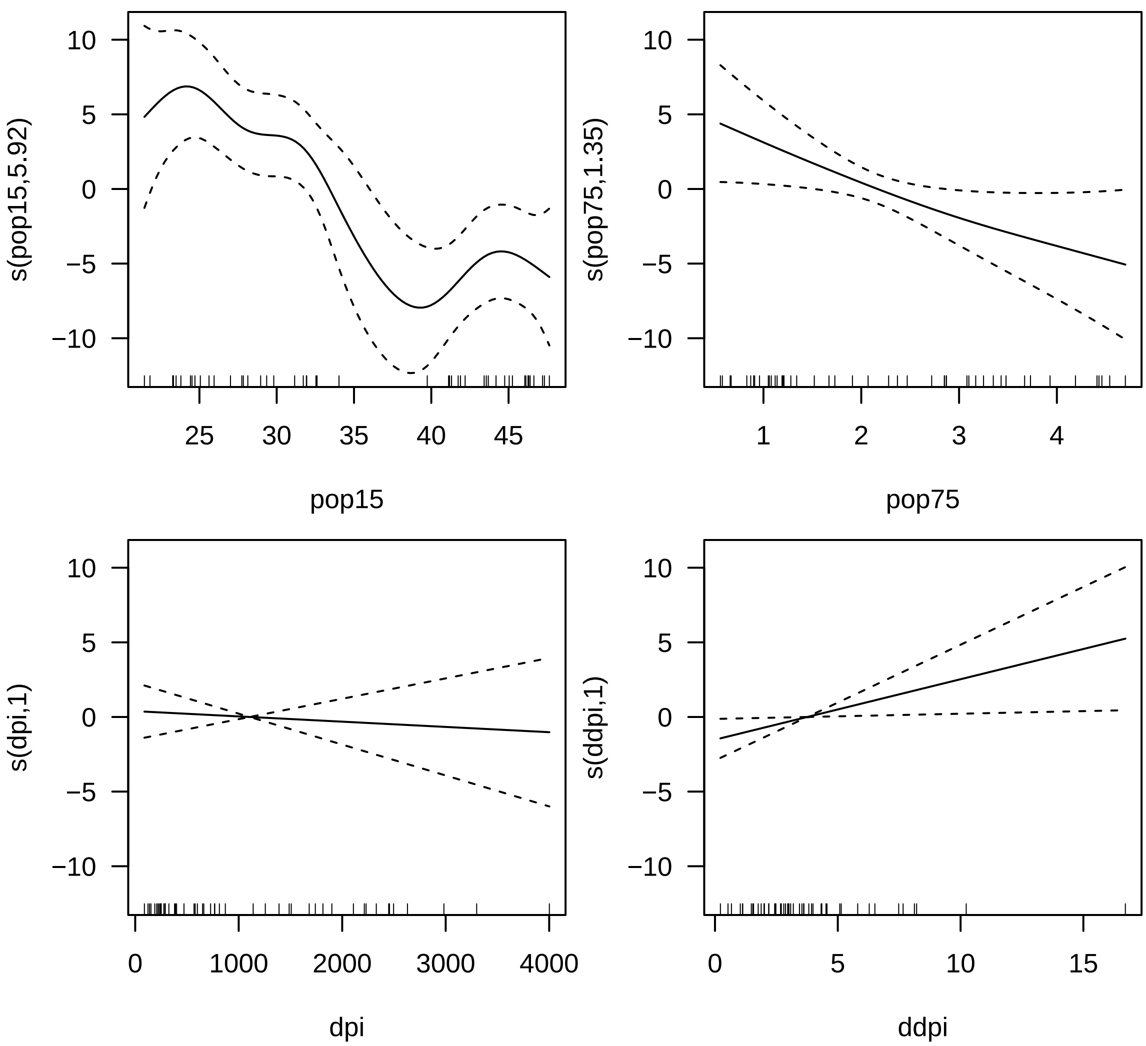
Generalized additive models for multiple predictors

```
library(faraway)
gamod <- gam(sr ~ s(pop15) + s(pop75) + s(dpi) + s(ddpi), data = savings)
summary(gamod)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## sr ~ s(pop15) + s(pop75) + s(dpi) + s(ddpi)
##
## Parametric coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   9.6710     0.4816   20.08   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##            edf Ref.df     F p-value
## s(pop15) 5.924  7.064 3.608 0.00406 **
## s(pop75) 1.350  1.623 2.811 0.06586 .
## s(dpi)   1.000  1.000 0.168 0.68403
```

```
## s(ddpi)  1.000  1.000 4.789 0.03455 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.422   Deviance explained = 53.2%
## GCV = 14.593  Scale est. = 11.595    n = 50
```

```r
par(mfrow = c(2, 2), mar = c(4, 3.85, 0.8, 0.5))
plot(gamod, las = 1)
```



Shrinkage Methods

The remainder of this R session is largely based on the R lab 'Ridge Regression and the Lasso' from the book 'Introduction to Statistical Learning with Applications in R' by Gareth James, Daniela Witten, Trevor Hastie, and Robert Tibshirani. We will use the `glmnet` package to perform ridge regression and the lasso to predict `Salary` on the `Hitters` data.

Ridge Regression

1. Data Setup

```r
library(ISLR)
data(Hitters)
Hitters = na.omit(Hitters)
head(Hitters)
```

```
##                    AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun
## -Alan Ashby          315   81     7   24  38    39    14   3449   835     69
## -Alvin Davis         479  130    18   66  72    76     3   1624   457     63
## -Andre Dawson        496  141    20   65  78    37    11   5628  1575    225
## -Andres Galarraga    321   87    10   39  42    30     2    396   101     12
## -Alfredo Griffin     594  169     4   74  51    35    11   4408  1133     19
## -Al Newman           185   37     1   23   8    21     2    214    42      1
##                    CRuns CRBI CWalks League Division PutOuts Assists Errors
## -Alan Ashby          321  414    375      N        W     632      43     10
## -Alvin Davis         224  266    263      A        W     880      82     14
## -Andre Dawson        828  838    354      N        E     200      11      3
## -Andres Galarraga     48   46     33      N        E     805      40      4
## -Alfredo Griffin     501  336    194      A        W     282     421     25
## -Al Newman            30    9     24      N        E      76     127      7
##                    Salary NewLeague
## -Alan Ashby         475.0         N
## -Alvin Davis        480.0         A
## -Andre Dawson       500.0         N
## -Andres Galarraga    91.5         N
## -Alfredo Griffin    750.0         A
## -Al Newman           70.0         A
```

```r
summary(Hitters)
```

```
##      AtBat            Hits           HmRun            Runs       
##  Min.   : 19.0   Min.   :  1.0   Min.   : 0.00   Min.   :  0.00  
##  1st Qu.:282.5   1st Qu.: 71.5   1st Qu.: 5.00   1st Qu.: 33.50  
##  Median :413.0   Median :103.0   Median : 9.00   Median : 52.00  
##  Mean   :403.6   Mean   :107.8   Mean   :11.62   Mean   : 54.75  
##  3rd Qu.:526.0   3rd Qu.:141.5   3rd Qu.:18.00   3rd Qu.: 73.00  
##  Max.   :687.0   Max.   :238.0   Max.   :40.00   Max.   :130.00  
##       RBI             Walks            Years            CAtBat       
##  Min.   :  0.00   Min.   :  0.00   Min.   : 1.000   Min.   :   19.0  
##  1st Qu.: 30.00   1st Qu.: 23.00   1st Qu.: 4.000   1st Qu.:  842.5  
##  Median : 47.00   Median : 37.00   Median : 6.000   Median : 1931.0  
##  Mean   : 51.49   Mean   : 41.11   Mean   : 7.312   Mean   : 2657.5  
##  3rd Qu.: 71.00   3rd Qu.: 57.00   3rd Qu.:10.000   3rd Qu.: 3890.5  
##  Max.   :121.00   Max.   :105.00   Max.   :24.000   Max.   :14053.0  
##      CHits            CHmRun           CRuns             CRBI       
##  Min.   :   4.0   Min.   :  0.00   Min.   :   2.0   Min.   :   3.0  
##  1st Qu.: 212.0   1st Qu.: 15.00   1st Qu.: 105.5   1st Qu.:  95.0  
##  Median : 516.0   Median : 40.00   Median : 250.0   Median : 230.0  
##  Mean   : 722.2   Mean   : 69.24   Mean   : 361.2   Mean   : 330.4  
##  3rd Qu.:1054.0   3rd Qu.: 92.50   3rd Qu.: 497.5   3rd Qu.: 424.5  
```

```
##  Max.    :4256.0   Max.   :548.00   Max.   :2165.0   Max.   :1659.0  
##      CWalks        League  Division    PutOuts          Assists     
##  Min.   :   1.0   A:139   E:129     Min.   :   0.0   Min.   :  0.0  
##  1st Qu.:  71.0   N:124   W:134     1st Qu.: 113.5   1st Qu.:  8.0  
##  Median : 174.0                     Median : 224.0   Median : 45.0  
##  Mean   : 260.3                     Mean   : 290.7   Mean   :118.8  
##  3rd Qu.: 328.5                     3rd Qu.: 322.5   3rd Qu.:192.0  
##  Max.   :1566.0                     Max.   :1377.0   Max.   :492.0  
##      Errors           Salary       NewLeague
##  Min.   : 0.000   Min.   :  67.5   A:141    
##  1st Qu.: 3.000   1st Qu.: 190.0   N:122    
##  Median : 7.000   Median : 425.0            
##  Mean   : 8.593   Mean   : 535.9            
##  3rd Qu.:13.000   3rd Qu.: 750.0            
##  Max.   :32.000   Max.   :2460.0            
```

```r
library(glmnet)
X <- model.matrix(Salary ~ ., data = Hitters)[, -1]
y <- Hitters$Salary
```

The `glmnet()` function has an `alpha` argument that determines what type of model is fit. If `alpha = 0` then a ridge regression model is fit, and if `alpha = 1` then a lasso model is fit. We first fit a ridge regression model, which minimizes

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{p}\beta_j^2,$$

where $\lambda \geq 0$ is a *tuning parameter* to be determined.

2. Fit Ridge Regression over a grid of λ values

```r
grid <- 10^seq(10, -2, length = 100)
ridge.mod <- glmnet(X, y, alpha = 0, lambda = grid)
```

3. Ridge Regression Coefficients

```r
dim(coef(ridge.mod))
```

```
## [1]  20 100
```

We expect the coefficient estimates to be much smaller, in terms of ℓ_2 norm, when a large value of λ is used.

```r
ridge.mod$lambda[50] #Display 50th lambda value
```

```
## [1] 11497.57
```

```r
coef(ridge.mod)[, 50] # Display coefficients associated with 50th lambda value
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs 
## 407.356050200   0.036957182   0.138180344   0.524629976   0.230701523 
##           RBI         Walks         Years        CAtBat         CHits 
```

```
##   0.239841459   0.289618741   1.107702929   0.003131815   0.011653637
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##   0.087545670   0.023379882   0.024138320   0.025015421   0.085028114
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
##  -6.215440973   0.016482577   0.002612988  -0.020502690   0.301433531
```

```
sqrt(sum(coef(ridge.mod)[-1, 50]^2)) # Calculate l2 norm
```

```
## [1] 6.360612
```

In contrast, here are the coefficients when $\lambda = 705$, along with their ℓ_2 norm. Note the much larger l_2 norm of the coefficients associated with this smaller value of λ.

```
ridge.mod$lambda[60] #Display 60th lambda value
```

```
## [1] 705.4802
```

```
coef(ridge.mod)[, 60] # Display coefficients associated with 60th lambda value
```

```
##  (Intercept)        AtBat         Hits        HmRun         Runs          RBI
##  54.32519950   0.11211115   0.65622409   1.17980910   0.93769713   0.84718546
##        Walks        Years       CAtBat        CHits       CHmRun        CRuns
##   1.31987948   2.59640425   0.01083413   0.04674557   0.33777318   0.09355528
##         CRBI       CWalks      LeagueN    DivisionW      PutOuts      Assists
##   0.09780402   0.07189612  13.68370191 -54.65877750   0.11852289   0.01606037
##       Errors   NewLeagueN
##  -0.70358655   8.61181213
```

```
sqrt(sum(coef(ridge.mod)[-1, 60]^2)) # Calculate l2 norm
```

```
## [1] 57.11001
```

We can use the `predict()` function for a number of purposes. For instance, we can obtain the ridge regression coefficients for a new value of λ, say 50:

```
predict(ridge.mod, s = 50, type = "coefficients")[1:20, ]
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
##  4.876610e+01 -3.580999e-01  1.969359e+00 -1.278248e+00  1.145892e+00
##           RBI         Walks         Years        CAtBat         CHits
##  8.038292e-01  2.716186e+00 -6.218319e+00  5.447837e-03  1.064895e-01
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##  6.244860e-01  2.214985e-01  2.186914e-01 -1.500245e-01  4.592589e+01
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
## -1.182011e+02  2.502322e-01  1.215665e-01 -3.278600e+00 -9.496680e+00
```

4. Training/Testing

We now split the samples into a training set and a test set in order to estimate the test error of ridge regression and later on the lasso.

```r
set.seed(1)
train <- sample(1:nrow(X), nrow(X) / 2)
test <- (-train)
y.test <- y[test]

# Fit Ridge regression to the training data
ridge.mod <- glmnet(X[train,], y[train], alpha = 0, lambda = grid, thresh = 1e-12)
# Predcict the salary to the testing data with lambda = 4
ridge.pred <- predict(ridge.mod, s = 4, newx = X[test,])
# Calculate the Root Mean Square Error (RMSE)
sqrt(mean((ridge.pred - y.test)^2))
```

```
## [1] 377.093
```

```r
# Compute the RMSE for the intercept-only model
sqrt(mean((mean(y[train]) - y.test)^2))
```

```
## [1] 473.9936
```

```r
# Change to a much larger lambda
ridge.pred <- predict(ridge.mod, s = 1e10, newx = X[test,])
sqrt(mean((ridge.pred - y.test)^2))
```

```
## [1] 473.9935
```

```r
# Change lambda to 0
ridge.pred <- predict(ridge.mod, s = 0, newx = X[test,])
sqrt(mean((ridge.pred - y.test)^2))
```

```
## [1] 409.6215
```

```r
lm(y ~ X, subset = train)
```

```
##
## Call:
## lm(formula = y ~ X, subset = train)
##
## Coefficients:
## (Intercept)       XAtBat        XHits       XHmRun        XRuns         XRBI
##    274.0145      -0.3521      -1.6377       5.8145       1.5424       1.1243
##      XWalks       XYears      XCAtBat       XCHits      XCHmRun       XCRuns
##      3.7287     -16.3773      -0.6412       3.1632       3.4008      -0.9739
##       XCRBI      XCWalks     XLeagueN   XDivisionW     XPutOuts     XAssists
##     -0.6005       0.3379     119.1486    -144.0831       0.1976       0.6804
##     XErrors  XNewLeagueN
##     -4.7128     -71.0951
```

```r
predict(ridge.mod, s = 0, type = "coefficients")[1:20,]
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs           RBI
##   274.2089049    -0.3699455    -1.5370022     5.9129307     1.4811980     1.0772844
##         Walks         Years        CAtBat         CHits        CHmRun         CRuns
##     3.7577989   -16.5600387    -0.6313336     3.1115575     3.3297885    -0.9496641
##          CRBI        CWalks       LeagueN     DivisionW       PutOuts       Assists
##    -0.5694414     0.3300136   118.4000592  -144.2867510     0.1971770     0.6775088
##        Errors    NewLeagueN
##    -4.6833775   -70.1616132
```

Instead of arbitrarily choosing $\lambda = 4$, it would be better to use cross-validation (CV) to choose the tuning parameter λ. We can do this using the built-in cross-validation function, `cv.glmnet()`. By default, the function performs 10-fold cross-validation, though this can be changed using the argument `folds`.

5. Cross-Validation (CV)

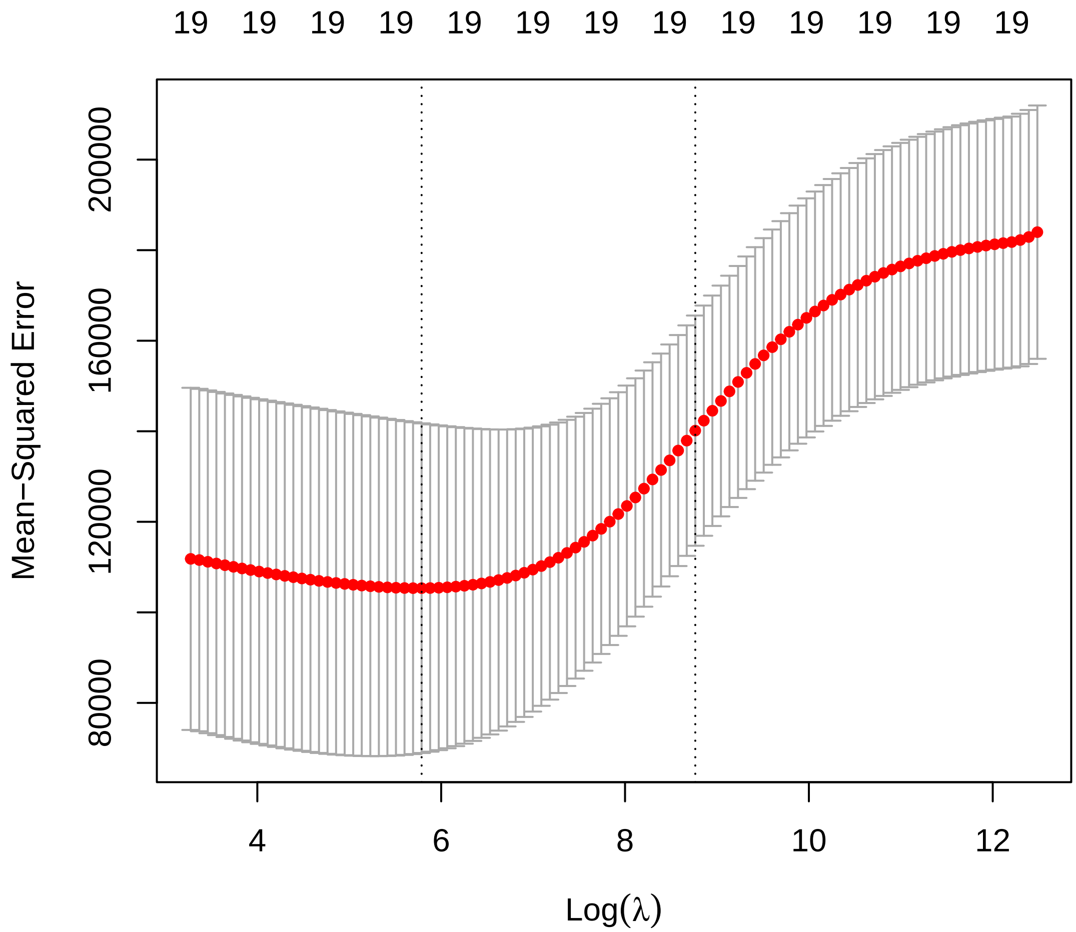
```r
set.seed(1)
# Fit ridge regression model on training data
cv.out <- cv.glmnet(X[train,], y[train], alpha = 0)
# Select lamda that minimizes training MSE
(bestLambda = cv.out$lambda.min)
```

```
## [1] 326.0828
```

```r
ridge.pred <- predict(ridge.mod, s = bestLambda, newx = X[test,])
sqrt(mean((ridge.pred - y.test)^2))
```

```
## [1] 373.9741
```

```r
plot(cv.out) # Draw plot of training MSE as a function of lambda
```



Finally, we refit our ridge regression model on the full data set, using the value of λ chosen by cross-validation, and examine the coefficient estimates.

```
# Fit ridge regression model on full dataset
out <- glmnet(X, y, alpha = 0)
# Display coefficients using lambda chosen by CV
predict(out, type = "coefficients", s = bestLambda)[1:20,]
```

```
##  (Intercept)        AtBat         Hits        HmRun         Runs          RBI
##  15.44383120   0.07715547   0.85911582   0.60103106   1.06369007   0.87936105
##        Walks        Years       CAtBat        CHits       CHmRun        CRuns
##   1.62444617   1.35254778   0.01134999   0.05746654   0.40680157   0.11456224
##         CRBI       CWalks      LeagueN    DivisionW      PutOuts      Assists
##   0.12116504   0.05299202  22.09143197 -79.04032656   0.16619903   0.02941950
##       Errors   NewLeagueN
##  -1.36092945   9.12487765
```

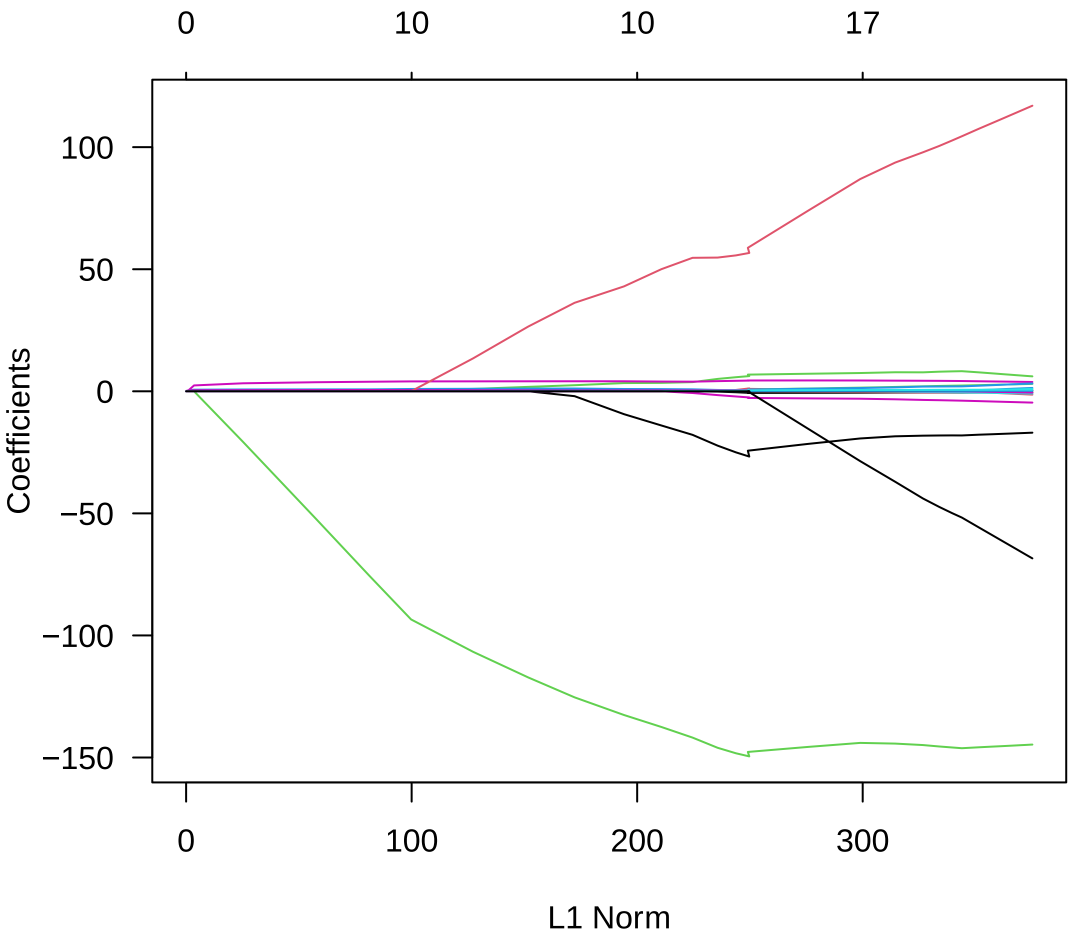
The Lasso

We saw that ridge regression with a wise choice of λ can outperform least squares as well as the null model on the Hitters data set. We now ask whether the lasso, which minimizes

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{p}|\beta_j|$$

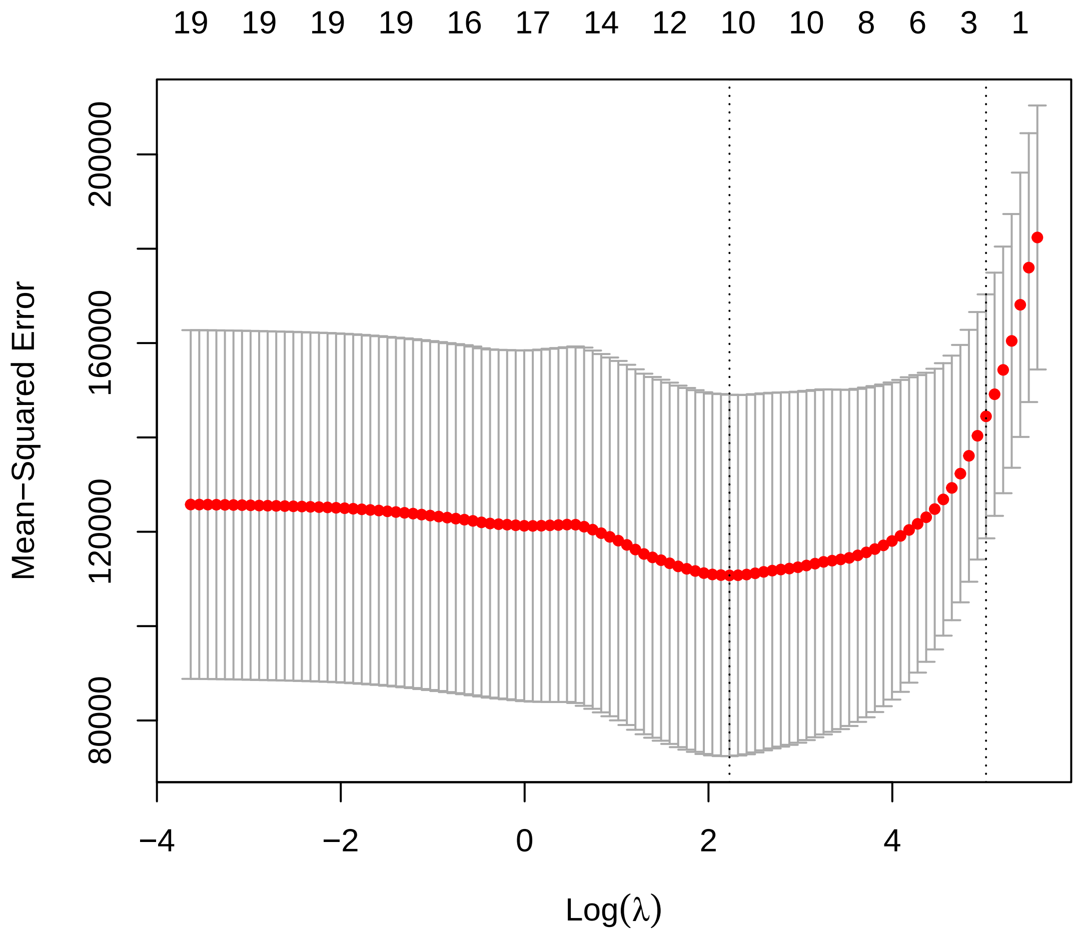
can yield either a more accurate or a more interpretable model than ridge regression. In order to fit a lasso model, we once again use the `glmnet()` function; however, this time we use the argument `alpha=1`.

```
# Fit lasso model on training data
lasso.mod <- glmnet(X[train,], y[train], alpha = 1, lambda = grid)
# Draw plot of coefficients
plot(lasso.mod, las = 1)
```



Notice that in the coefficient plot that depending on the choice of tuning parameter, some of the coefficients are exactly equal to zero. We now perform cross-validation and compute the associated test error:

```
set.seed(1)
# Fit lasso model on training data
cv.out <- cv.glmnet(X[train,], y[train], alpha = 1)
# Draw plot of training MSE as a function of lambda
plot(cv.out)
```




```r
# Select lamda that minimizes training MSE
bestLambda <- cv.out$lambda.min
# Use best lambda to predict test data
lasso.pred <- predict(lasso.mod, s = bestLambda, newx = X[test,])
# Calculate test RMSE
sqrt(mean((lasso.pred - y[test])^2))
```

```
## [1] 379.043
```

This is substantially lower than the test set RMSE of the null model and of least squares, and very similar to the test RMSE of ridge regression with λ chosen by cross-validation.

However, the lasso has a substantial advantage over ridge regression in that the resulting coefficient estimates are sparse. Here we see that 8 of the 19 coefficient estimates are exactly zero:

```r
# Fit lasso model on full dataset
out <- glmnet(X, y, alpha = 1, lambda = grid)
# Display coefficients using lambda chosen by CV
(lasso.coef <- predict(out, type = "coefficients", s = bestLambda)[1:20,])
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
##    1.27479059   -0.05497143    2.18034583    0.00000000    0.00000000
##           RBI         Walks         Years        CAtBat         CHits
```

```
##     0.00000000    2.29192406   -0.33806109    0.00000000    0.00000000
##         CHmRun         CRuns          CRBI        CWalks       LeagueN
##     0.02825013    0.21628385    0.41712537    0.00000000   20.28615023
##      DivisionW       PutOuts       Assists        Errors    NewLeagueN
## -116.16755870    0.23752385    0.00000000   -0.85629148    0.00000000
```

```
lasso.coef[lasso.coef != 0] # Display only non-zero coefficients
```

```
##   (Intercept)         AtBat          Hits         Walks         Years
##    1.27479059   -0.05497143    2.18034583    2.29192406   -0.33806109
##        CHmRun         CRuns          CRBI       LeagueN     DivisionW
##    0.02825013    0.21628385    0.41712537   20.28615023 -116.16755870
##       PutOuts        Errors
##    0.23752385   -0.85629148
```