Lecture 8

Introduction to Design of Experiments

Reading: Oehlert 2010 Chapters 1, 2; DAE 2017 Chapters 1, 2

DSA 8020 Statistical Methods II

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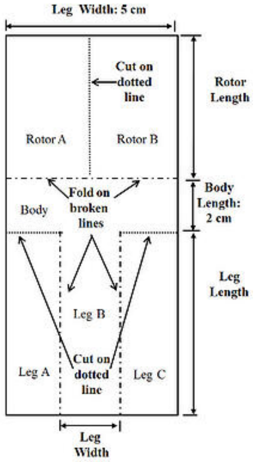
Agenda

1. **Background and Definitions**
2. **Fundamental Principles of Experimental Design**
3. **History of Experimental Design**

Paper Helicopter Experiment

Suppose we want to investigate how long a paper helicopter can fly in the air.



Source: https://blog.minitab.com/en/learning-design-of-experiments-with-paper-helicopters-and-minitab

Some of potential determining factors:

- Paper type (light, medium, and heavy)
- Rotor length (7.5cm or 8.5cm)
- Leg length (7.5cm or 12 cm)
- Leg width (3.2cm or 5cm)

Steps for Planning, Conducting and Analyzing an Experiment

- State the problem of interest
- Select the response variable and determine the factor(s)
- Choose the design and conduct the experiment
- Perform statistical analysis
- Draw conclusions

Battery Experiment [DAE 2017 p. 24]

- **Specific question**: How do battery types vary with respect to life-per-unit cost?
- **Response**: Time (per unit cost) to exhaust battery under standard load
- **Comparative**: Difference between 4 battery types
- **Controlled**: All compared using the same device
- **Replication**: Four batteries of each type tested

Some Definitions

- **Factor:** Variable whose influence upon the response variable. Settings of factor are called **levels**
- **Treatments:** The procedures (a set of values for all factors) used for comparison
- **Experimental units:** Objects on which treatments are applied
- **Measurement units:** Objects on which the response is measured. These may differ from the experimental units
- **Randomization:** Using a known probabilistic mechanism to assign treatments to experimental units
- **Experimental error:** Variation in response outcomes (modeled as random)

Main Elements of An Experiment

An experiment applies treatments to experimental units and measures responses.

- Want to learn about **treatments** (e.g., dose of drug; nano-tech coating for a fabric)
- **Responses** tell us how the treatment worked (patient get better; stain resistance)
- Experimenter assigns treatments to **experimental units** (e.g., a patient; a bolt of fabric)

Experimental Unit

- Perhaps the most important concept in statistical design
- The experimental unit is the unit (subject, plant, pot, animal) which is randomly assigned to a treatment
- The experimental unit *defines the unit to be replicated to increase degrees of freedom*

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If a group of "units" must have the same treatment, they are likely measurement units (MUs) rather than experimental units (EUs)

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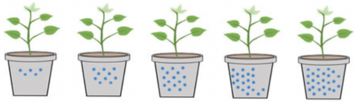

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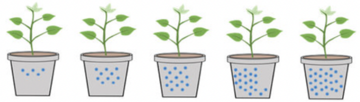
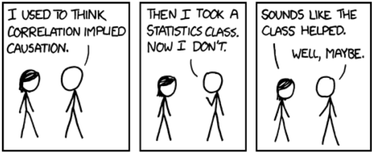


- Different food placed in tanks containing the fish. Fish are not the EUs

Observational vs. Experimental Studies

- An observational study has the same triple of treatment, unit, and response, but one **observes** the assignment of treatments to units (e.g., human health studies on cigarette smoke and adverse health effects)
- What makes an experimental study special is **control**. The experimenter gets to control the assignment of treatments to the experimental units
- Experiments can make causal inference while observational studies find association



Source: Slide 5 at `http://users.stat.umn.edu/~gary/classes/5303/lectures/Introduction.pdf`

Why Designed Experiments?

- Design for direct comparison of treatments
- Design to reduce bias in comparisons (avoid systematic errors)
- Design to reduce variability (be precise)
- Experiments support causual inference

Fundamental Principles: Replication, Randomization, and Blocking

Replication

- Each treatment is applied to (experimental) units that are representative of the population
- Enable the estimation/quantification of **experimental error** using standard deviation
- Decrease variance of estimates and increase the power to detect significant differences: for independent $y_i's$,

$$\mathrm{Var}(\frac{1}{n}\sum_{i=1}^{n} y_i) = \frac{1}{n}\mathrm{Var}(y_1)$$

Randomization

Use of a chance mechanism such as random number generators to assign treatments to (experimental) units. It has the following advantages:

- Protect against latent variables or “lurking” variables
- Reduce influence of subjective bias in treatment assignments (e.g., clinical trials)
- Ensure validity of statistical inference

Blocking

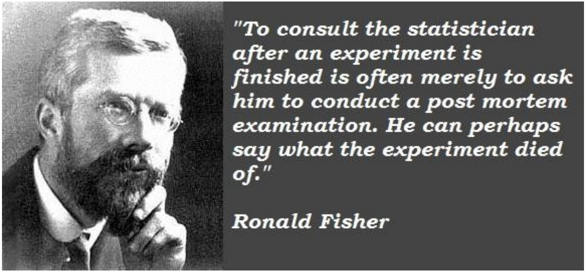
A **block** refers to a collection of homogeneous units. Effective blocking: larger between-block variations than within-block variations.

Examples: hours, batches, lots.

- Run and compare treatments within the same blocks to eliminate block-block variation and reduce variability of treatment effects estimates
- Block what you can and randomize what you cannot

A Brief History of Experimental Design: Agricultural Era

- R. A. Fisher and his co-workers, Rothamsted Agricultural Experimental Station (1930, England)
- Introduced statistical experimental design and data analysis. Summarized the fundamental principles: **replication**, **randomization**, and **blocking**
- Factorial designs, ANOVA



A Brief History of Experimental Design: Industrial Eras

- The first industrial era, 1951 - late 1970s
  - Process modeling and optimization
  - G. E. P. Box & K. B. Wilson and coworkers in chemical industries and other processing industries
  - Empirical modeling, response surface methodologies, central composite design
- The second industrial era, late 1970s - 1990
  - Quality improvement and variation reduction
  - G. Taguchi and robust parameter design

A Brief History of Experimental Design: Modern Era

- Popular outside statistics, and an indispensable tool in many scientific/engineering endeavors
- New challenges:
  - Large and complex experiments, e.g., screening design in pharmaceutical industry, experimental design in biotechnology
  - **Computer experiments:** efficient ways to model complex systems based on computer simulation
  - ...

Summary

These slides cover:

- Basic concepts of design of experiments (DOE):
- A brief history of DOE
- Fundamental principles: randomization, blocking, replication

Plans for the Next Three Weeks

- Completely Randomized Designs
- Randomized Complete Block Designs, Factorial Designs, and Split-Plot Designs
- Random and Mixed Effects Models
- Computer Experiments