# MATH 4070 R Session 5: Autocorrelation and Time Series Models

Whitney

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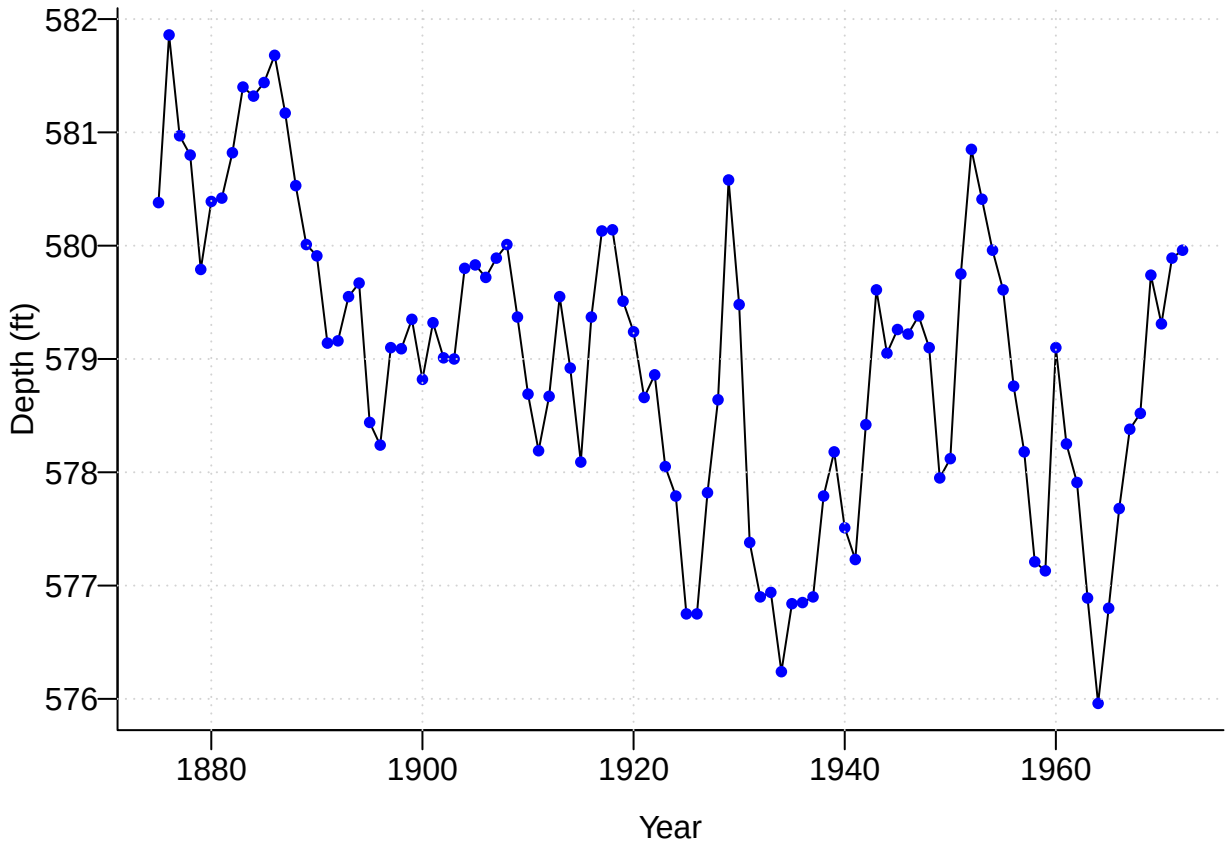
## Time Series Data

### Lake Huron Time Series

Annual measurements of the level of Lake Huron in feet

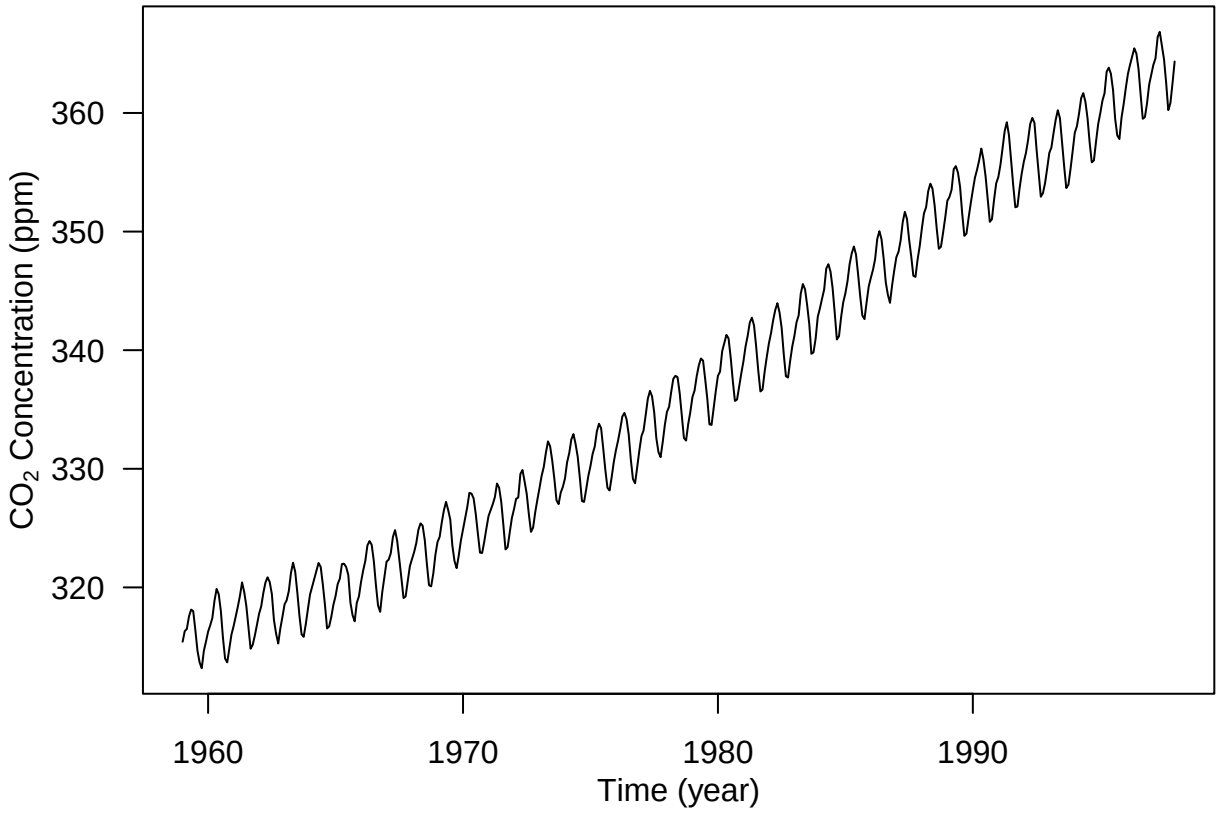
```
par(mar = c(3.2, 3.2, 0.5, 0.5), mgp = c(2, 0.5, 0), bty = "L")
data(LakeHuron)
plot(LakeHuron, ylab = "Depth (ft)", xlab = "Year", las = 1)
points(LakeHuron, cex = 0.8, col = "blue", pch = 16)
grid()
```




### $CO_2$ Concentration

Atmospheric concentrations of CO2 are expressed in parts per million (ppm) and reported in the preliminary 1997 SIO manometric mole fraction scale.

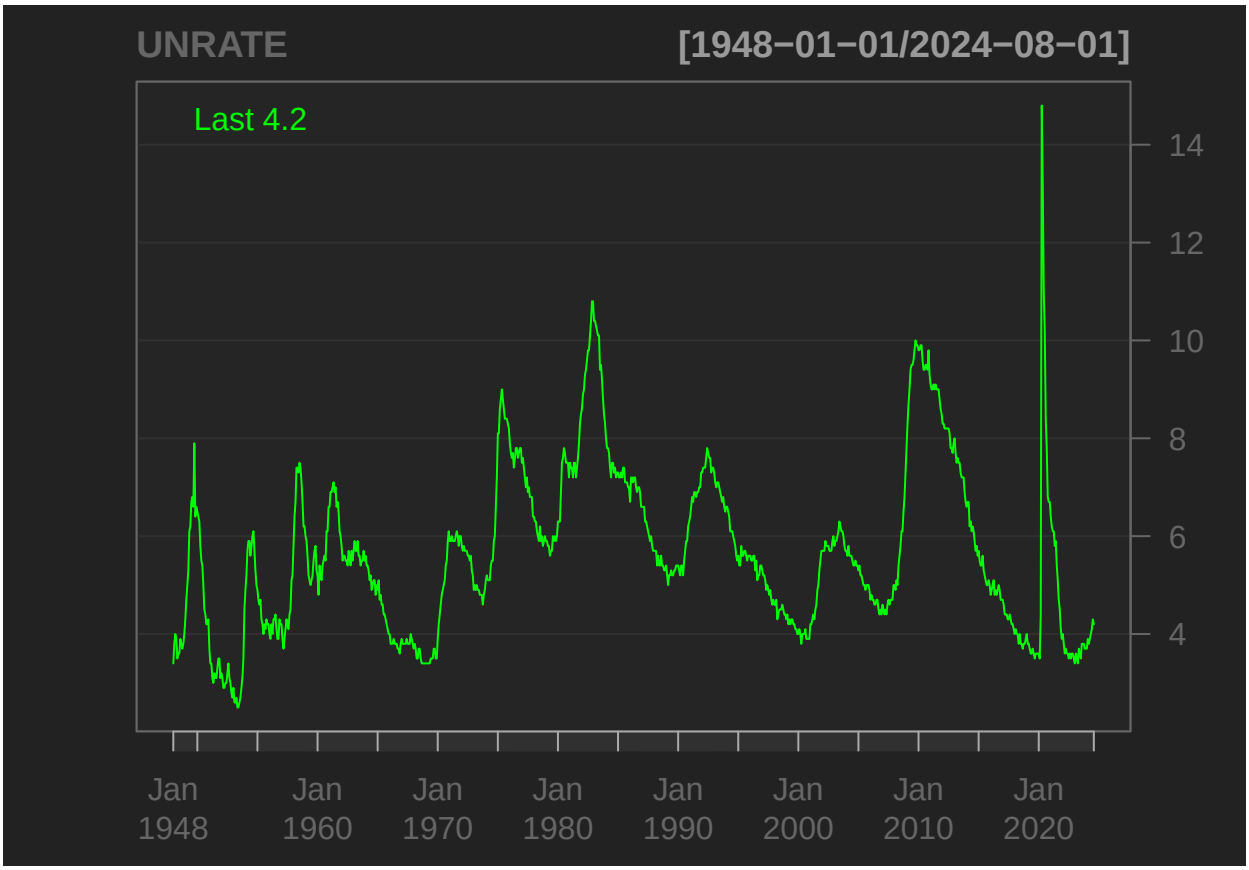
```
data(co2)
par(mar = c(3.8, 4, 0.8, 0.6))
plot(co2, las = 1, xlab = "", ylab = "")
mtext("Time (year)", side = 1, line = 2)
mtext(expression(paste("CO"[2], " Concentration (ppm)")), side = 2, line = 2.5)
```




**U.S. monthly unemployment rates**

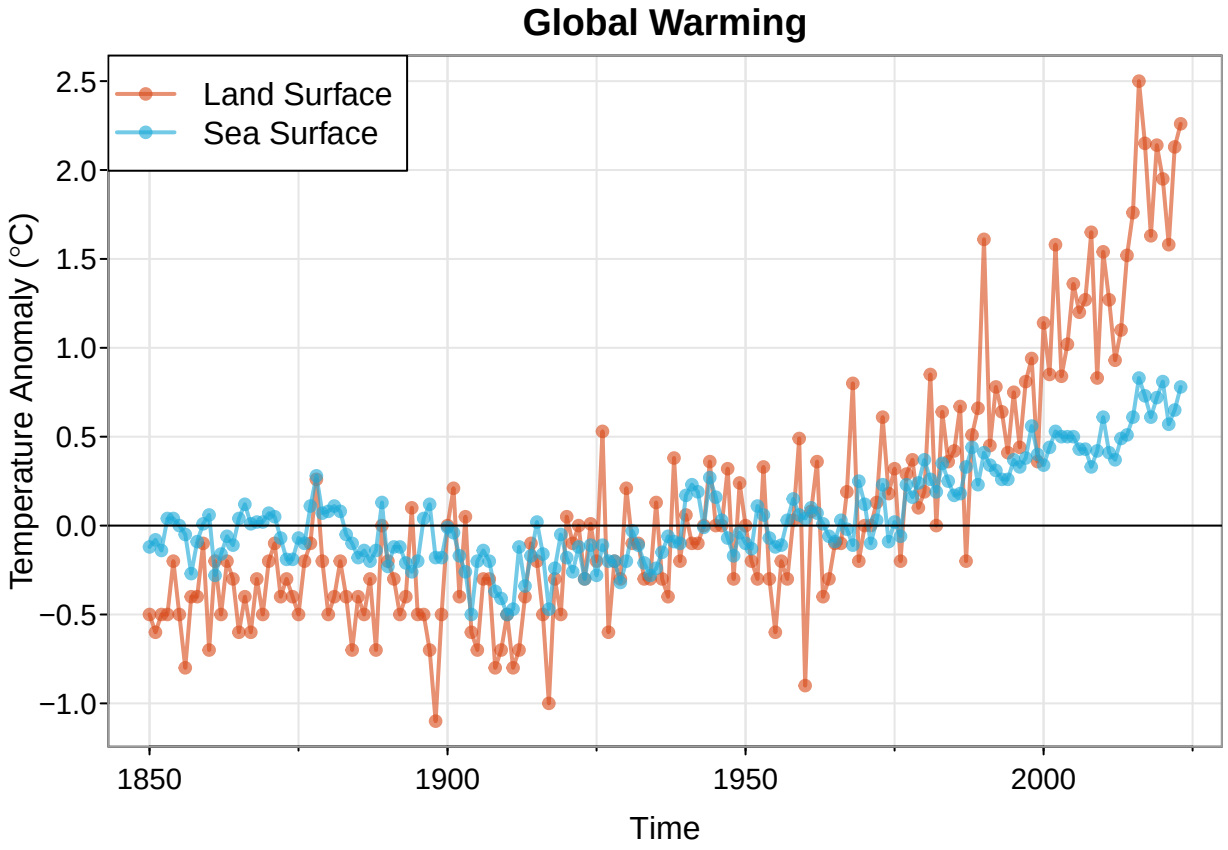
```
library(quantmod)
getSymbols("UNRATE", src = "FRED"); chartSeries(UNRATE)
```

```
## [1] "UNRATE"
```



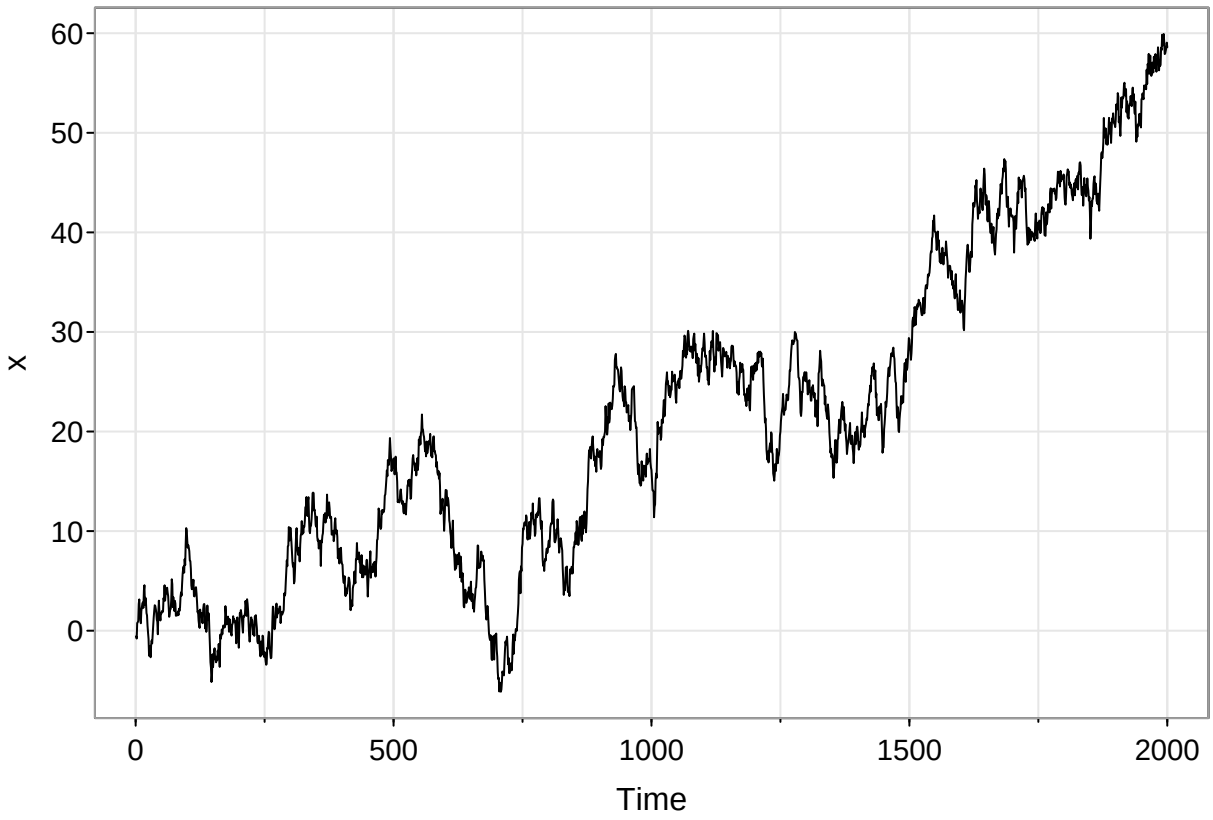

**Global mean land temperature anomalies**

```
library(astsa)
culer = c(rgb(217, 77, 30, 160, max = 255), rgb(30, 170, 217, 160, max = 255))
tsplot(gtemp_land, col = culer[1], lwd = 2, type = "o", pch = 20, las = 1,
ylab = expression(paste("Temperature Anomaly (", degree, "C)")), main = "Global Warming")
lines(gtemp_ocean, col = culer[2], lwd = 2, type = "o", pch = 20)
abline(h = 0)
legend("topleft", col = culer, lty = 1, lwd = 2, pch = 20,
       legend = c("Land Surface", "Sea Surface"), bg = "white")
```




**Simulated time series**

```
set.seed(123)
w <- rnorm(2000); x <- cumsum(w); tsplot(x, las = 1)
```




## Autocorrelation

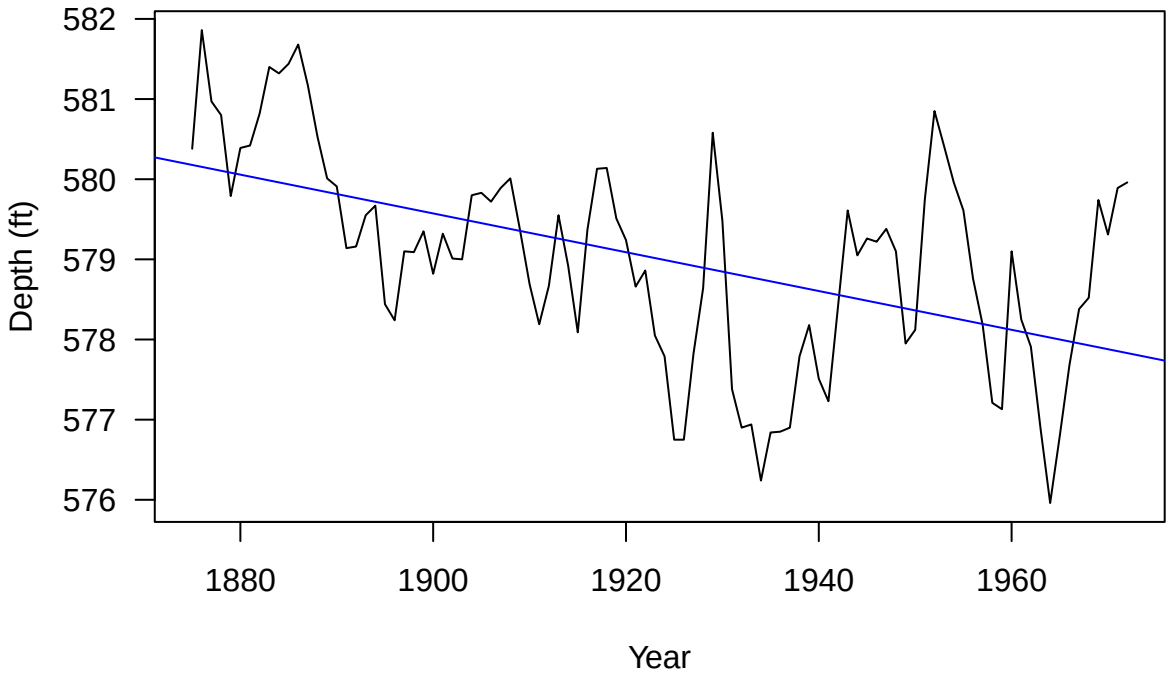
### Trend estimation

```
library(astsa)
yr <- 1875:1972
lm <- lm(LakeHuron ~ yr)
summary(lm)
```

```
##
## Call:
## lm(formula = LakeHuron ~ yr)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.50997 -0.72726  0.00083  0.74402  2.53565
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 625.554918   7.764293  80.568  < 2e-16 ***
## yr           -0.024201   0.004036  -5.996 3.55e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.13 on 96 degrees of freedom
```

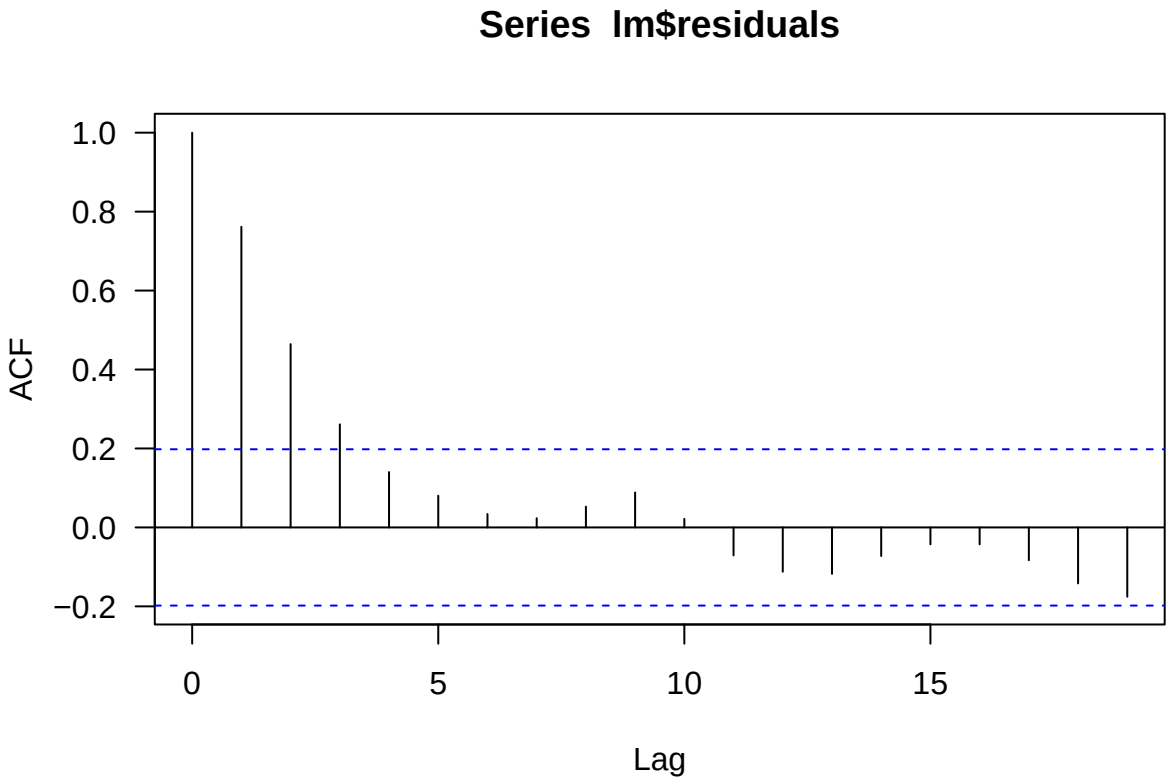
```
## Multiple R-squared:  0.2725, Adjusted R-squared:  0.2649
## F-statistic: 35.95 on 1 and 96 DF,  p-value: 3.545e-08
```

```r
plot(LakeHuron, ylab = "Depth (ft)", xlab = "Year", las = 1)
abline(lm, col = "blue")
```




**Sample autocorrelation function**

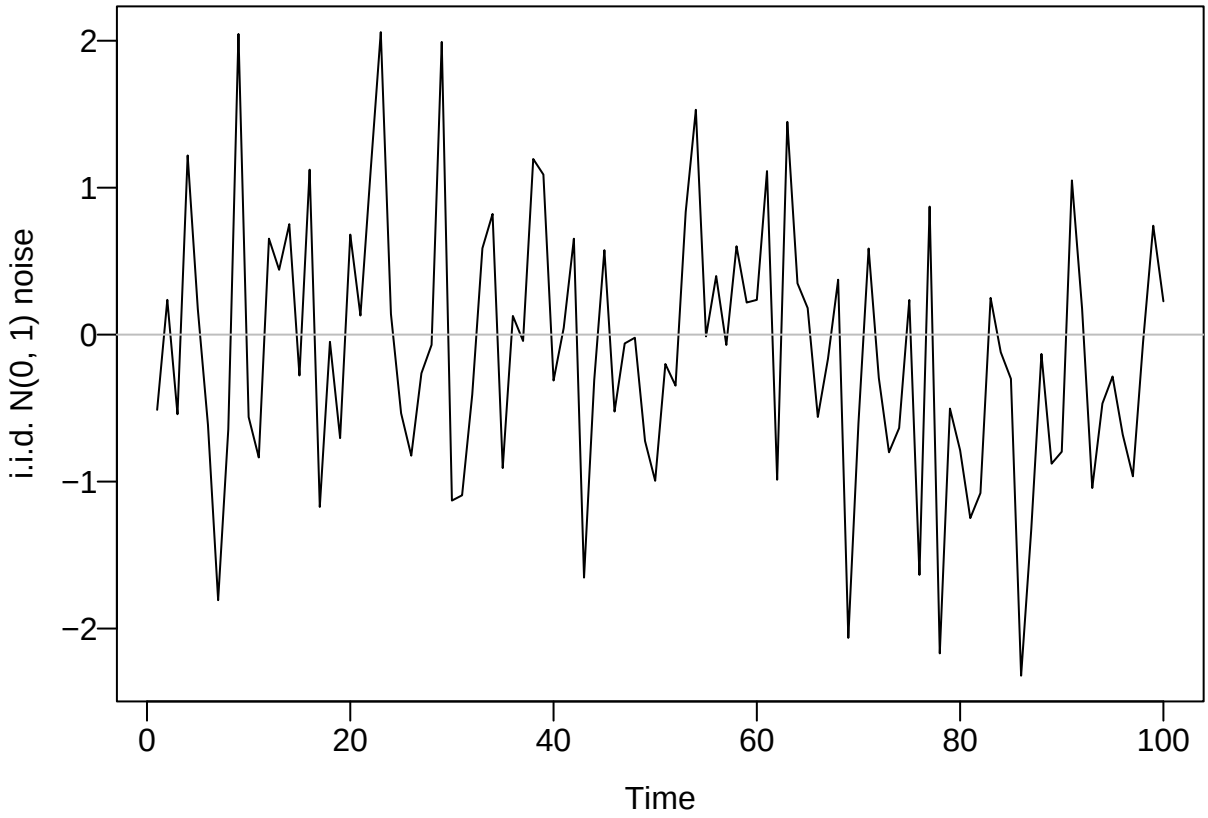
```r
acf(lm$residuals, las = 1)
```



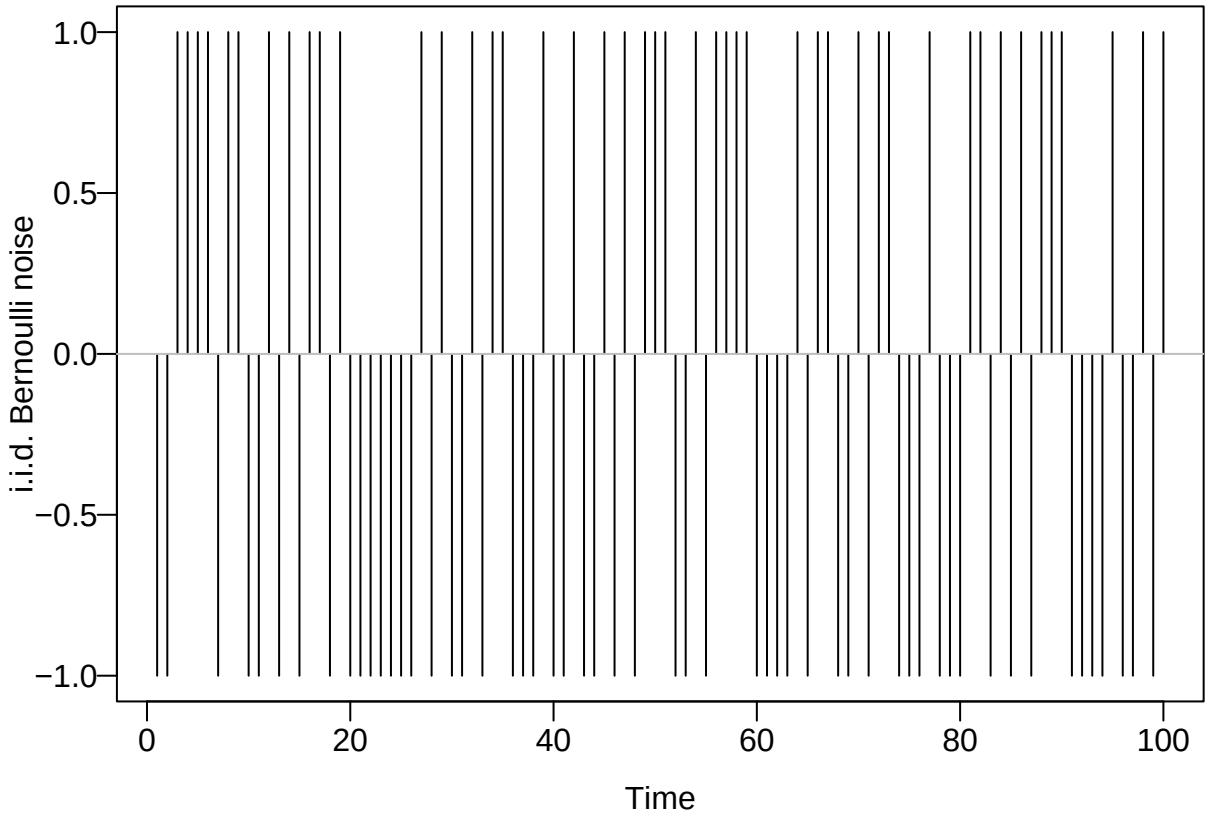

### Examples of i.i.d. Noise

```
T = 100
t <- 1:T

## i.i.d. normal
normal_iid <- rnorm(T)
par(las = 1, mgp = c(2, 0.5, 0), mar = c(3.6, 3.6, 0.8, 0.6))
plot(t, normal_iid, type = "l", las = 1,
     xlab = "Time", ylab = "i.i.d. N(0, 1) noise")
abline(h = 0, col = "gray")
```




```
## i.i.d. Binary
ber_iid <- replicate(T, rbinom(1, 1, 0.5))
ber_iid <- ifelse(ber_iid == 0, -1, 1)
plot(t, ber_iid, type = "h", las = 1,
     xlab = "Time", ylab = "i.i.d. Bernoulli noise")
abline(h = 0, col = "gray")
```


1.0
0.5
0.0
−0.5
−1.0
i.i.d. Bernoulli noise
0
20
40
60
80
100
Time