MATH 8090: ARMA I

Whitney Huang, Clemson University

9/19-9/21/2023

Contents

Checking Stationarity and Causality

Consider the following AR(4) process

$$\eta_t = 2.7607\eta_{t-1} - 3.8106\eta_{t-2} + 2.6535\eta_{t-3} - 0.9238\eta_{t-4} + Z_t,$$

the AR characteristic polynomial is

$$\phi(z) = 1 - 2.7607z + 3.8106z^2 - 2.6535z^3 + 0.9238z^4$$

```
# calculate the roots of the polynomial, and store in xi
xi <- polyroot(c(1, -2.7607, 3.8106, -2.6535, 0.9238))
xi
```

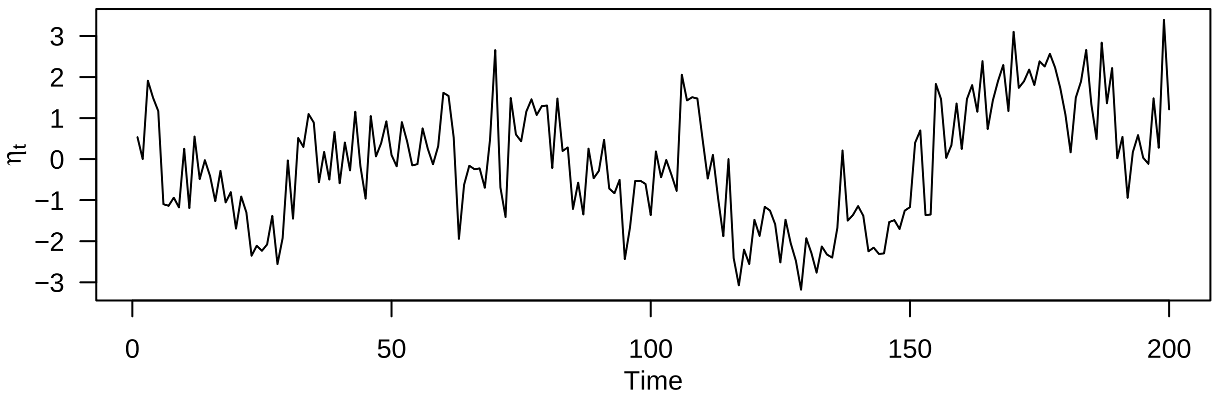
```
## [1] 0.6499635+0.7859373i 0.7862240+0.6500408i 0.6499635-0.7859373i
## [4] 0.7862240-0.6500408i
```

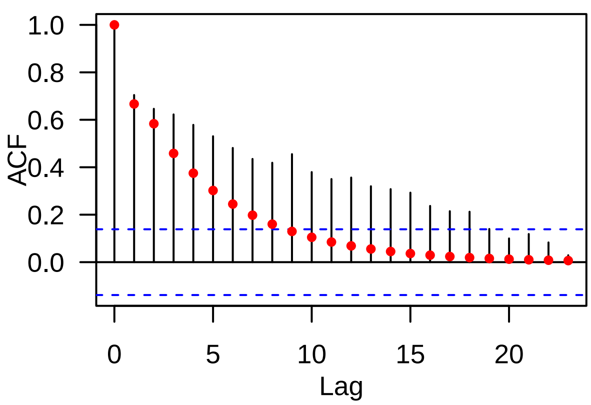
```
# calculate the modulus of the roots, zetas
Mod(xi)
```

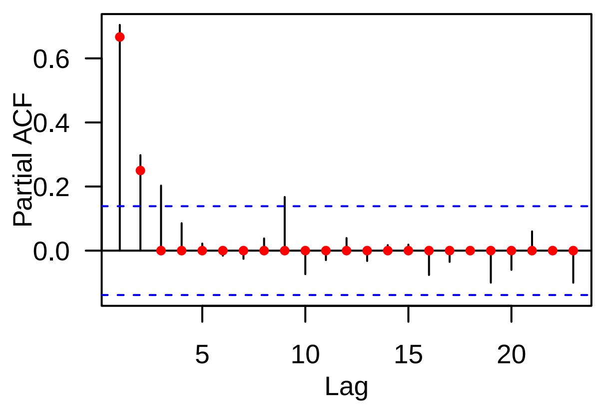
```
## [1] 1.019877 1.020148 1.019877 1.020148
```

PACF

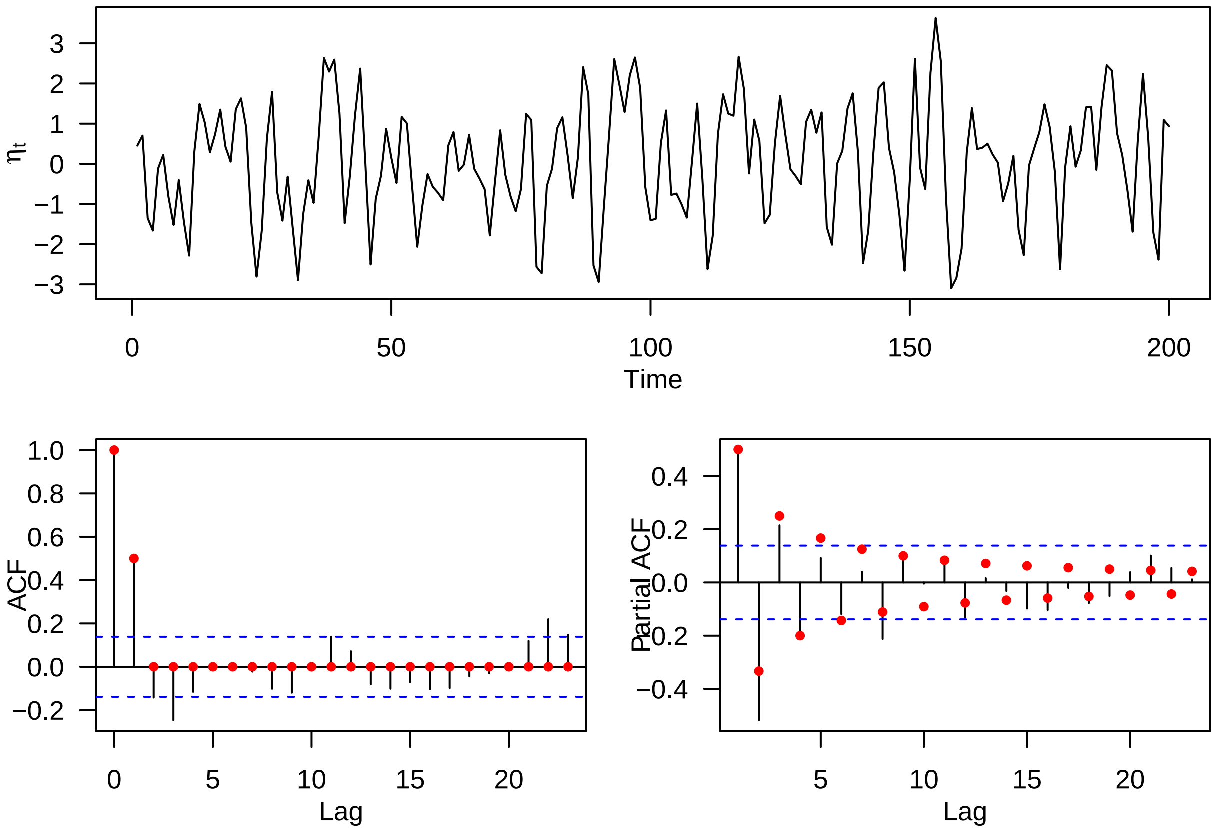
```r
ar2_1 <- arima.sim(n = 200, list(ar = c(0.5, 0.25)))

par(las = 1, mgp = c(2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
layout(matrix(c(1, 1, 2, 3), 2, 2, byrow = TRUE))
ts.plot(ar2_1, ylab = expression(eta[t]))
acf(ar2_1)
acf_true <- ARMAacf(ar = c(0.5, 0.25), lag.max = 23)
points(0:23, acf_true, pch = 16, cex = 0.8, col = "red")
pacf(ar2_1)
pacf_true <- ARMAacf(ar = c(0.5, 0.25), lag.max = 23, pacf = T)
points(1:23, pacf_true, pch = 16, cex = 0.8, col = "red")
```



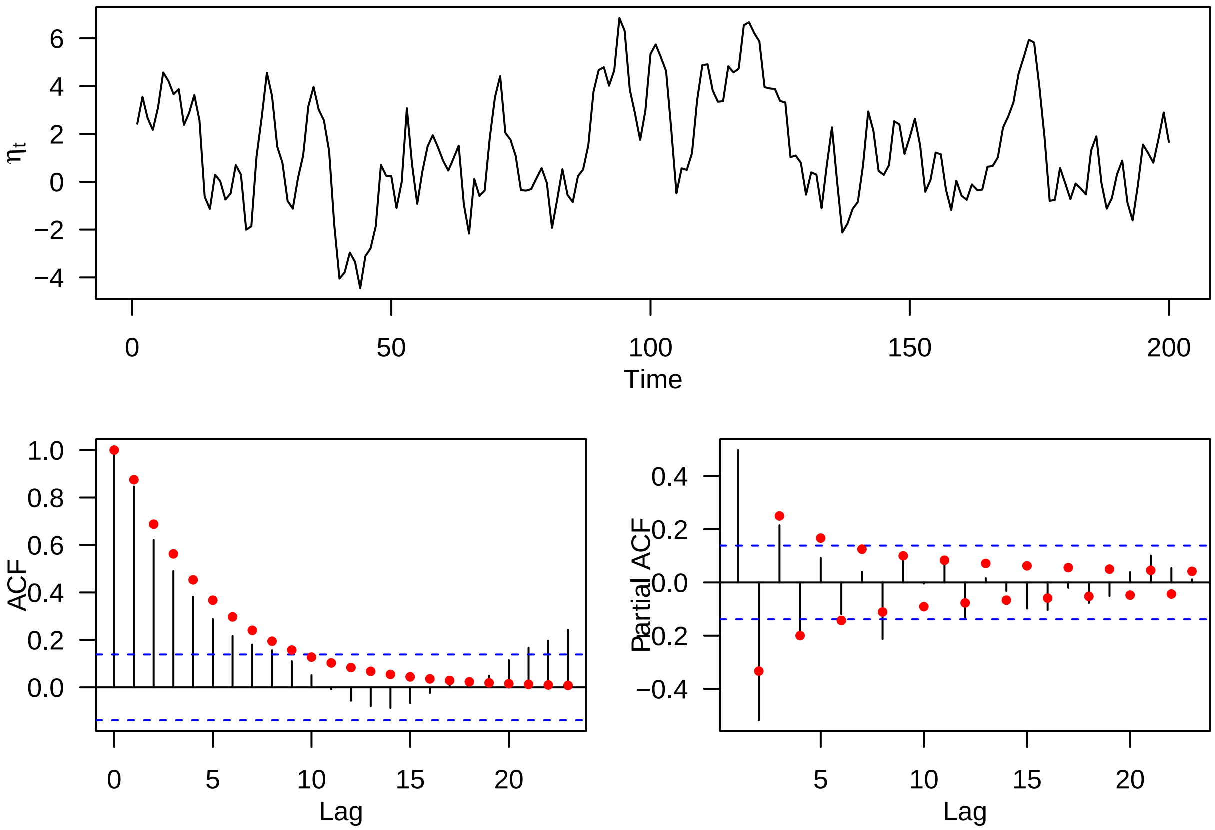





```r
ma1 <- arima.sim(n = 200, list(ma = c(1)))

par(las = 1, mgp = c(2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
layout(matrix(c(1, 1, 2, 3), 2, 2, byrow = TRUE))
ts.plot(ma1, ylab = expression(eta[t]))
acf(ma1)
acf_true <- ARMAacf(ma = c(1), lag.max = 23)
points(0:23, acf_true, col = "red", cex = 0.8, pch = 16)
pacf(ma1)
pacf_true <- ARMAacf(ma = c(1), lag.max = 23,
                     pacf = T)
points(1:23, pacf_true, col = "red", cex = 0.8, pch = 16)
```

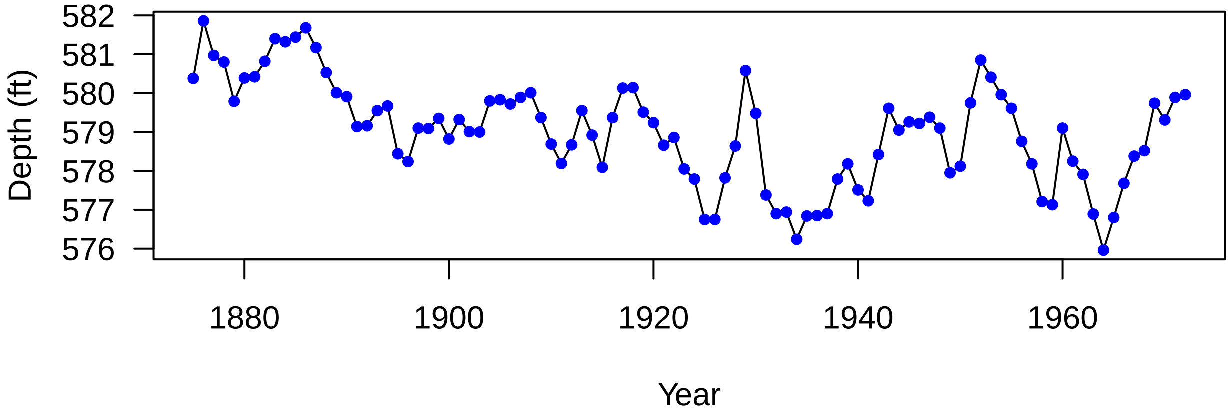


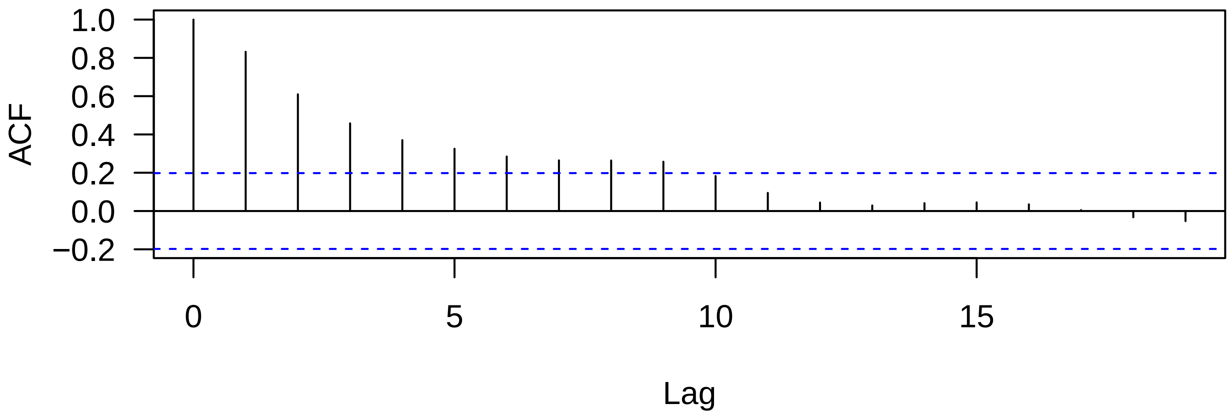

```
arma <- arima.sim(n = 200, list(ar = c(0.5, 0.25), ma = c(1)))

par(las = 1, mgp = c(2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
layout(matrix(c(1, 1, 2, 3), 2, 2, byrow = TRUE))
ts.plot(arma, ylab = expression(eta[t]))
acf(arma)
acf_true <- ARMAacf(ar = c(0.5, 0.25), ma = c(1), lag.max = 23)
points(0:23, acf_true, col = "red", cex = 0.8, pch = 16)
pacf(ma1)
pacf_true <- ARMAacf(ar = c(0.5, 0.25), ma = c(1), lag.max = 23,
                     pacf = T)
points(1:23, pacf_true, col = "red", cex = 0.8, pch = 16)
```



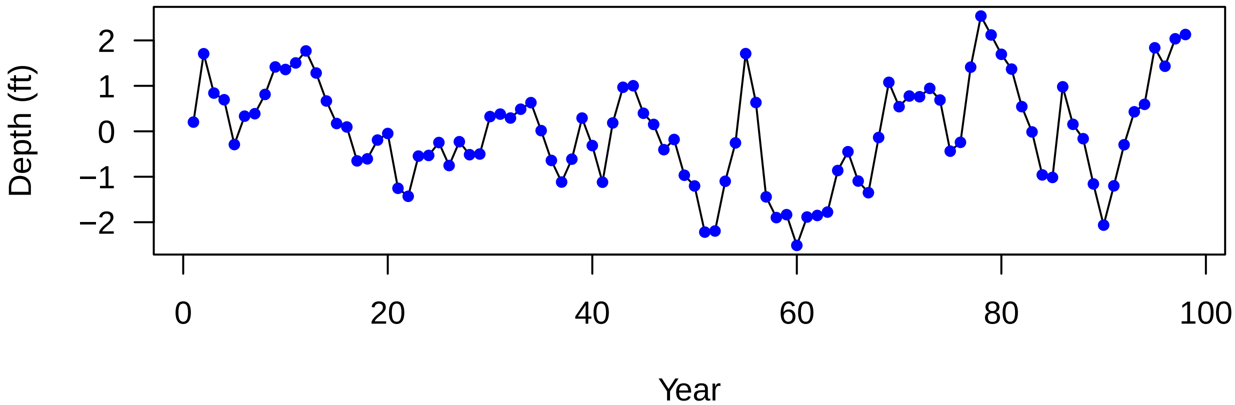
Lake Huron Example

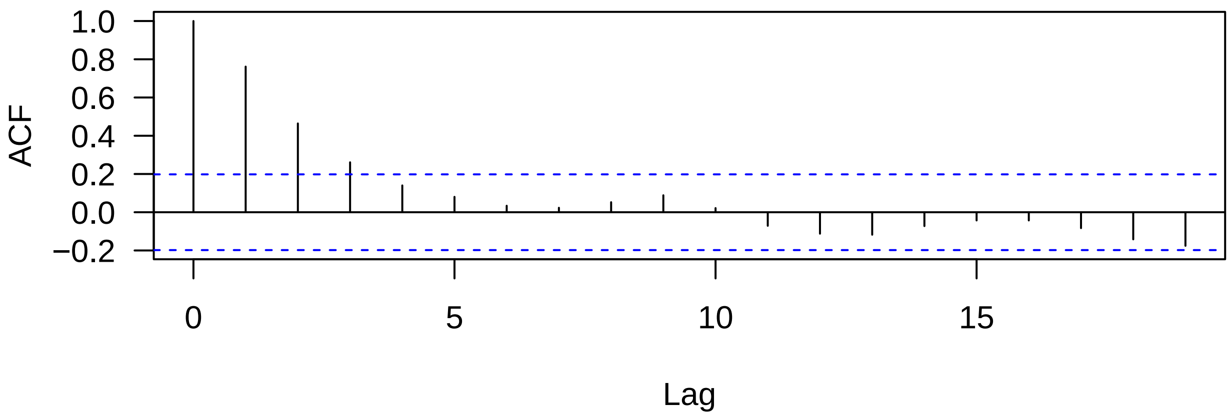
```
data(LakeHuron)
par(las = 1, mfrow = c(2, 1), mar = c(4, 4, 0.8, 0.6))
plot(LakeHuron, ylab = "Depth (ft)", xlab = "Year")
points(LakeHuron, cex = 0.8, col = "blue", pch = 16)
acf(LakeHuron)
```



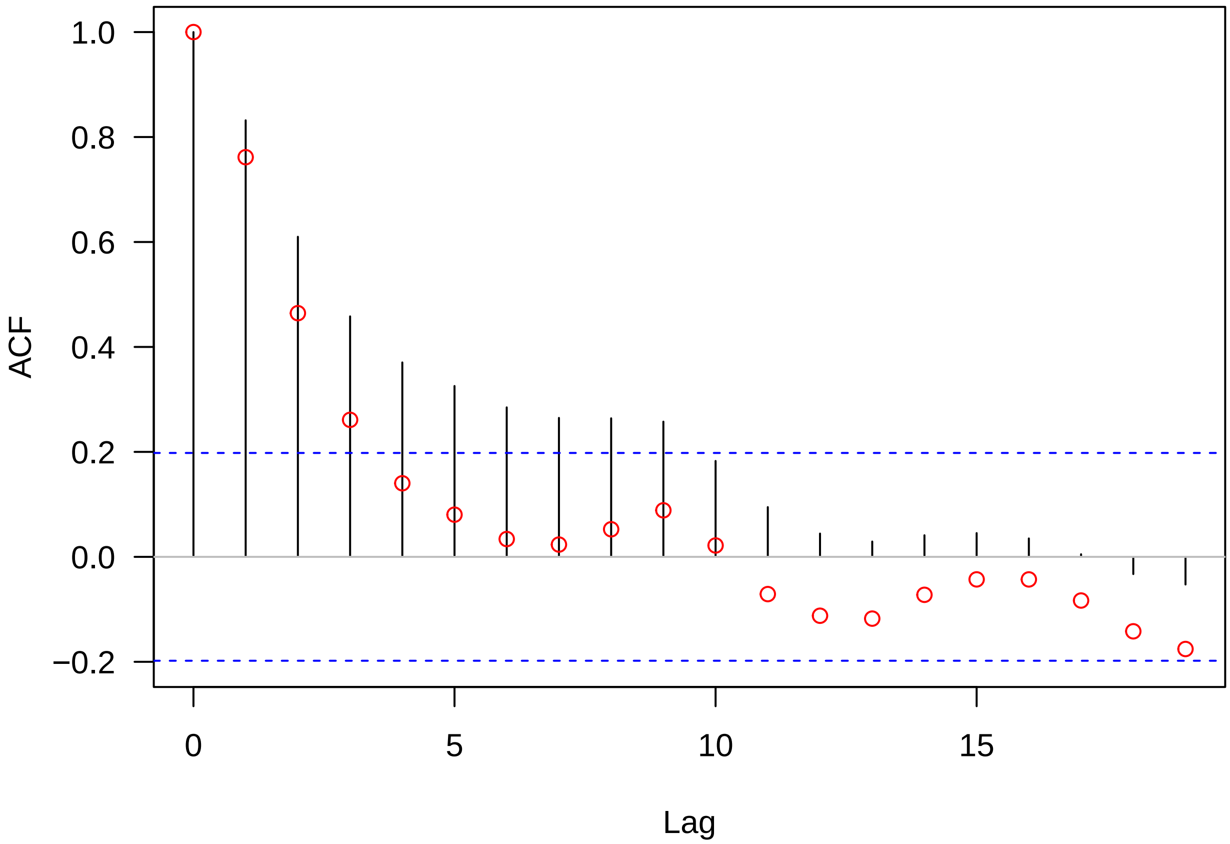



```r
# Let's remove the (linear trend)
yr <- 1875:1972
lm <- lm(LakeHuron ~ yr)
plot(lm$residuals, ylab = "Depth (ft)", xlab = "Year", type = "l")
points(lm$residuals, cex = 0.8, col = "blue", pch = 16)
acf(lm$residuals)
```

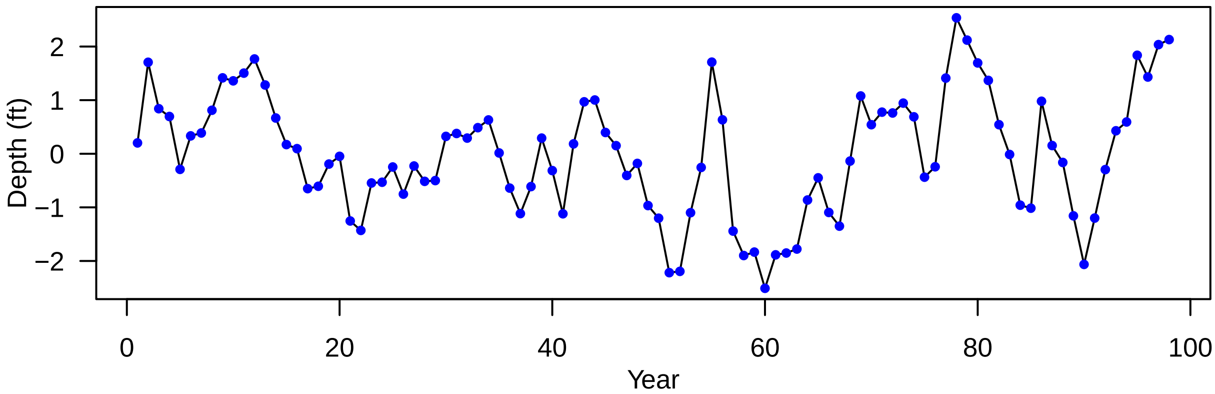


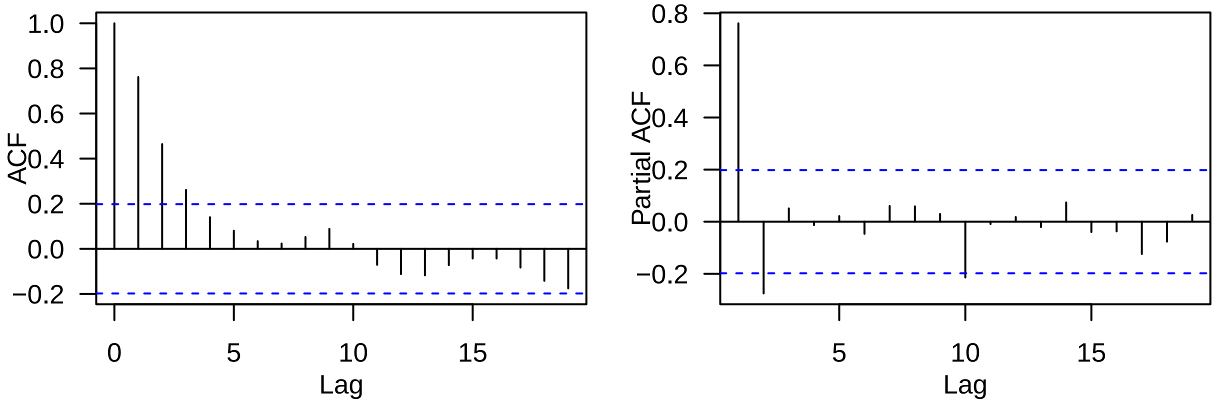



```
par(mfrow = c(1, 1), las = 1)
plot(0:19, acf(LakeHuron, plot = F)$acf, type = "h", xlab = "Lag", ylab = "ACF", ylim = c(-0.2, 1))
abline(h = 0, col = "gray")
abline(h = c(-1, 1) * qnorm(0.975) / sqrt(length(LakeHuron)) , col = "blue", lty = 2)
acf_detrend <- acf(lm$residuals, plot = F)$acf
points(0:19, acf_detrend, col = "red")
```




```
par(las = 1, mgp = c(2, 1, 0), mar = c(3.6, 3.6, 0.8, 0.6))
layout(matrix(c(1, 1, 2, 3), 2, 2, byrow = TRUE))
plot(lm$residuals, ylab = "Depth (ft)", xlab = "Year", type = "l")
points(lm$residuals, cex = 0.8, col = "blue", pch = 16)
acf(lm$residuals)
pacf(lm$residuals)
```

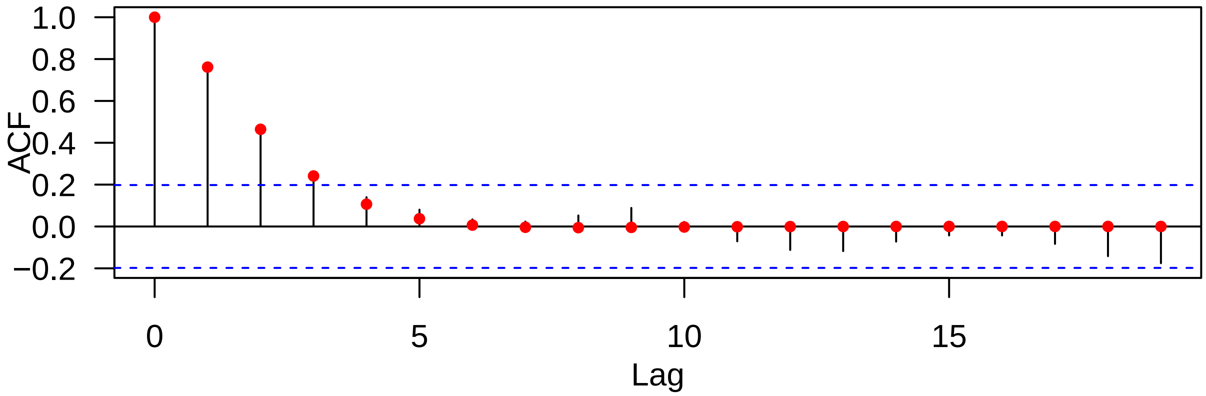


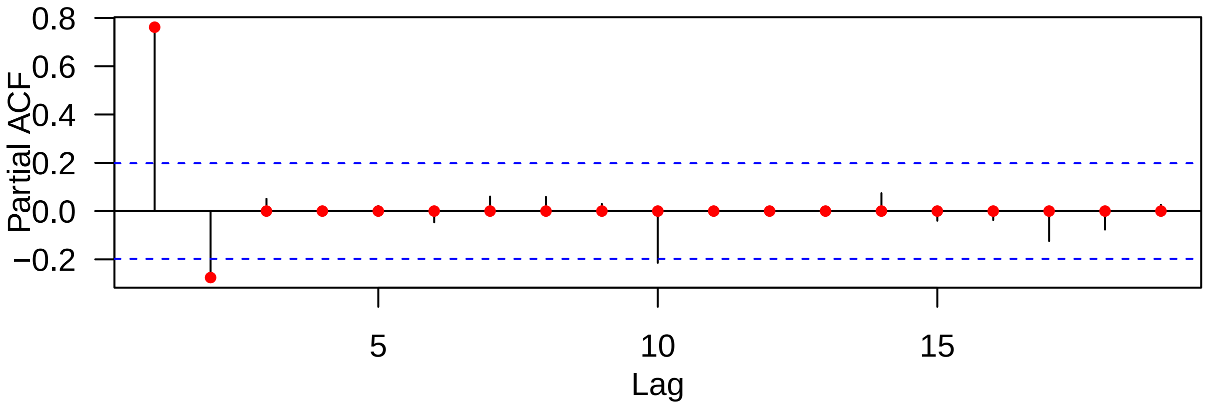


An example of Yule-Walker estimate

Let's fit an AR(2) to the detrended lake huron series

```
YW_est <- ar(lm$residuals, aic = F, order.max = 2, method = "yw")
# plot sample and estimated acf/pacf
par(las = 1, mgp = c(2, 1, 0), mar = c(3.6, 3.6, 0.6, 0.6), mfrow = c(2, 1))
acf(lm$residuals)
acf_YWest <- ARMAacf(ar = YW_est$ar, lag.max = 23)
points(0:23, acf_YWest, col = "red", pch = 16, cex = 0.8)
pacf(lm$residuals)
pacf_YWest <- ARMAacf(ar = YW_est$ar, lag.max = 23, pacf = T)
points(1:23, pacf_YWest, col = "red", pch = 16, cex = 0.8)
```





MLE

```
(MLE_est1 <- arima(lm$residuals, order = c(2, 0, 0), include.mean = F))
```

```
##
## Call:
## arima(x = lm$residuals, order = c(2, 0, 0), include.mean = F)
##
## Coefficients:
##          ar1      ar2
##       1.0050  -0.2925
## s.e.  0.0976   0.1002
##
## sigma^2 estimated as 0.4572:  log likelihood = -101.26,  aic = 208.51
```

```
(MLE_est2 <- arima(lm$residuals, order = c(1, 0, 0), include.mean = F))
```

```
##
## Call:
## arima(x = lm$residuals, order = c(1, 0, 0), include.mean = F)
##
## Coefficients:
##          ar1
##       0.7826
```

```
## s.e.  0.0635
##
## sigma^2 estimated as 0.4975:  log likelihood = -105.32,  aic = 214.65
```

```r
(MLE_est3 <- arima(lm$residuals, order = c(2, 0, 1), include.mean = F))
```

```
##
## Call:
## arima(x = lm$residuals, order = c(2, 0, 1), include.mean = F)
##
## Coefficients:
##          ar1      ar2     ma1
##       0.8381  -0.1631  0.1842
## s.e.  0.3178   0.2618  0.3179
##
## sigma^2 estimated as 0.4556:  log likelihood = -101.09,  aic = 210.19
```

```r
(MLE_est4 <- arima(LakeHuron, order = c(2, 0, 0), xreg = yr))
```

```
##
## Call:
## arima(x = LakeHuron, order = c(2, 0, 0), xreg = yr)
##
## Coefficients:
##          ar1      ar2  intercept       yr
##       1.0048  -0.2913   620.5115  -0.0216
## s.e.  0.0976   0.1004    15.5771   0.0081
##
## sigma^2 estimated as 0.4566:  log likelihood = -101.2,  aic = 212.4
```

Diagnostic

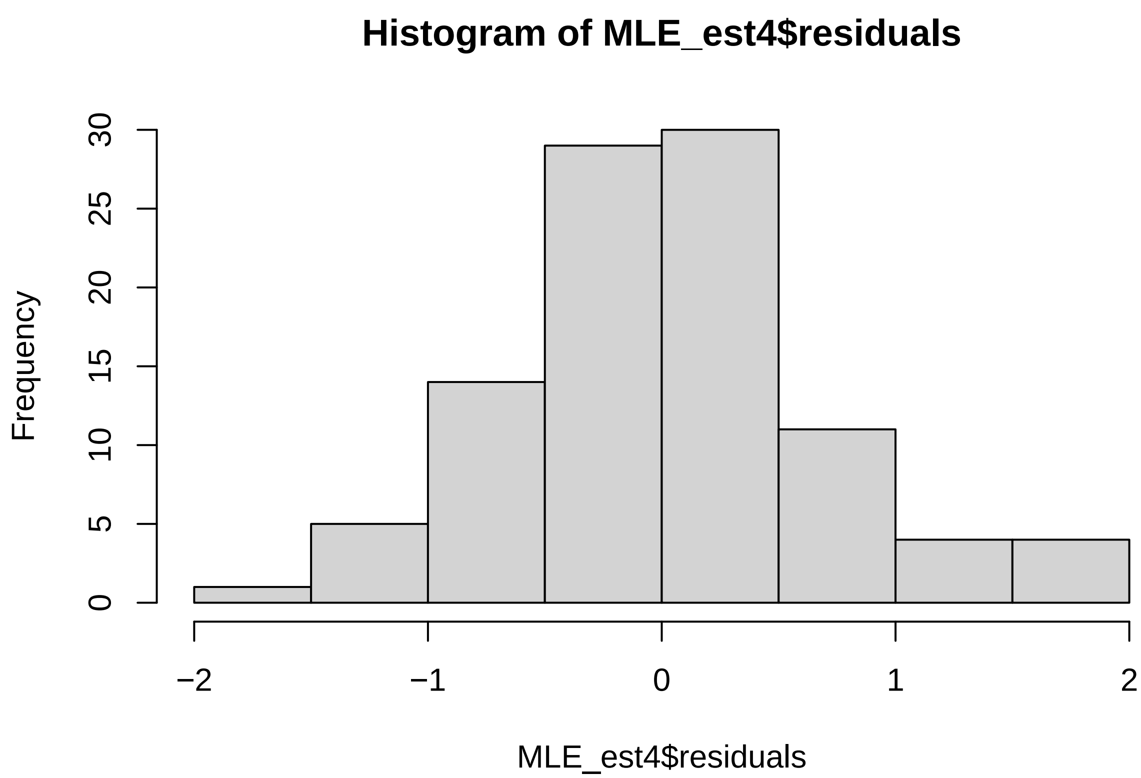
```r
Box.test(YW_est$resid[-(1:2)], type = "Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  YW_est$resid[-(1:2)]
## X-squared = 0.56352, df = 1, p-value = 0.4528
```

```r
Box.test(MLE_est4$residuals, type = "Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  MLE_est4$residuals
## X-squared = 0.03358, df = 1, p-value = 0.8546
```

```r
hist(MLE_est4$residuals)
```




```r
qqnorm(MLE_est4$residuals, col = "blue", cex = 0.8)
qqline(MLE_est4$residuals)
```