Lecture 20

Categorical Data Analysis III

Readings: IntroStat Chapter 10

STAT 8010 Statistical Methods I

June 14, 2023

Whitney Huang

Clemson University

Agenda

1. **Contingency Table: Test for Independence**
2. **Fisher's Exact Test**

Example

A 2011 study was conducted in Kalamazoo, Michigan. The objective was to determine if parents' marital status affects children's marital status later in their life. In total, 2,000 children were interviewed. The columns refer to the parents' marital status. Use the contingency table below to conduct a χ^2 test from beginning to end. Use $\alpha = .10$

(Observed)	Married	Divorced	Total
Married	581	487	
Divorced	455	477	
Total			

Example Cont'd

1. Define the Null and Alternative hypotheses:

   H_0 : there is no relationship between parents' marital status and childrens' marital status.

   H_a : there is a relationship between parents' marital status and childrens' marital status

2. Calculate the marginal totals, and the grand total

(Observed)	Married	Divorced	Total
Married	581	487	1068
Divorced	455	477	932
Total	1036	964	2000

Example Cont'd

3. Calculate the expected cell counts

(Expected)	Married	Divorced
Married	$\frac{1068\times1036}{2000} = 553.224$	$\frac{1068\times964}{2000} = 514.776$
Divorced	$\frac{932\times1036}{2000} = 482.776$	$\frac{932\times964}{2000} = 449.224$

Example Cont'd

3. Calculate the expected cell counts

(Expected)	Married	Divorced
Married	$\frac{1068\times1036}{2000} = 553.224$	$\frac{1068\times964}{2000} = 514.776$
Divorced	$\frac{932\times1036}{2000} = 482.776$	$\frac{932\times964}{2000} = 449.224$

4. Calculate the partial χ^2 values

partial χ^2	Married	Divorced
Married	$\frac{(581-553.224)^2}{553.224} = 1.39$	$\frac{(487-514.776)^2}{514.776} = 1.50$
Divorced	$\frac{(455-482.776)^2}{482.776} = 1.60$	$\frac{(477-449.224)^2}{449.224} = 1.72$

Example Cont'd

3. Calculate the expected cell counts

(Expected)	Married	Divorced
Married	$\frac{1068\times1036}{2000}=553.224$	$\frac{1068\times964}{2000}=514.776$
Divorced	$\frac{932\times1036}{2000}=482.776$	$\frac{932\times964}{2000}=449.224$

4. Calculate the partial χ^2 values

partial χ^2	Married	Divorced
Married	$\frac{(581-553.224)^2}{553.224}=1.39$	$\frac{(487-514.776)^2}{514.776}=1.50$
Divorced	$\frac{(455-482.776)^2}{482.776}=1.60$	$\frac{(477-449.224)^2}{449.224}=1.72$

5. Calculate the χ^2 statistic

$$\chi^2 = 1.39 + 1.50 + 1.60 + 1.72 = 6.21$$

Example Cont'd

3. Calculate the expected cell counts

(Expected)	Married	Divorced
Married	$\frac{1068\times1036}{2000} = 553.224$	$\frac{1068\times964}{2000} = 514.776$
Divorced	$\frac{932\times1036}{2000} = 482.776$	$\frac{932\times964}{2000} = 449.224$

4. Calculate the partial χ^2 values

partial χ^2	Married	Divorced
Married	$\frac{(581-553.224)^2}{553.224} = 1.39$	$\frac{(487-514.776)^2}{514.776} = 1.50$
Divorced	$\frac{(455-482.776)^2}{482.776} = 1.60$	$\frac{(477-449.224)^2}{449.224} = 1.72$

5. Calculate the χ^2 statistic
$\chi^2 = 1.39 + 1.50 + 1.60 + 1.72 = 6.21$

6. Calculate the degrees of freedom (df)
The df is $(2-1)\times(2-1) = 1$

Example Cont'd

3. Calculate the expected cell counts

(Expected)	Married	Divorced
Married	$\frac{1068\times1036}{2000} = 553.224$	$\frac{1068\times964}{2000} = 514.776$
Divorced	$\frac{932\times1036}{2000} = 482.776$	$\frac{932\times964}{2000} = 449.224$

4. Calculate the partial χ^2 values

partial χ^2	Married	Divorced
Married	$\frac{(581-553.224)^2}{553.224} = 1.39$	$\frac{(487-514.776)^2}{514.776} = 1.50$
Divorced	$\frac{(455-482.776)^2}{482.776} = 1.60$	$\frac{(477-449.224)^2}{449.224} = 1.72$

5. Calculate the χ^2 statistic
   $\chi^2 = 1.39 + 1.50 + 1.60 + 1.72 = 6.21$
6. Calculate the degrees of freedom (df)
   The df is $(2 - 1) \times (2 - 1) = 1$
7. Find the χ^2 critical value with respect to α from the χ^2 table
   The $\chi^2_{\alpha=0.1,df=1} = 2.71$

Example Cont'd

3. Calculate the expected cell counts

(Expected)	Married	Divorced
Married	$\frac{1068\times1036}{2000} = 553.224$	$\frac{1068\times964}{2000} = 514.776$
Divorced	$\frac{932\times1036}{2000} = 482.776$	$\frac{932\times964}{2000} = 449.224$

4. Calculate the partial χ^2 values

partial χ^2	Married	Divorced
Married	$\frac{(581-553.224)^2}{553.224} = 1.39$	$\frac{(487-514.776)^2}{514.776} = 1.50$
Divorced	$\frac{(455-482.776)^2}{482.776} = 1.60$	$\frac{(477-449.224)^2}{449.224} = 1.72$

5. Calculate the χ^2 statistic
   $\chi^2 = 1.39 + 1.50 + 1.60 + 1.72 = 6.21$
6. Calculate the degrees of freedom (df)
   The df is $(2-1)\times(2-1) = 1$
7. Find the χ^2 critical value with respect to α from the χ^2 table
   The $\chi^2_{\alpha=0.1,df=1} = 2.71$
8. Draw your conclusion:
   We reject H_0 and conclude that there is a relationship between parents' marital status and childrens' marital status.

Example

The following contingency table contains enrollment data for a random sample of students from several colleges at Purdue University during the 2006-2007 academic year. The table lists the number of male and female students enrolled in each college. Use the two-way table to conduct a χ^2 test from beginning to end. Use $\alpha = .01$

(Observed)	Female	Male	Total
Liberal Arts	378	262	640
Science	99	175	274
Engineering	104	510	614
Total	581	947	1528

Example Cont'd

(Expected)	Female	Male
Liberal Arts	$\frac{640\times581}{1528} = 243.35$	$\frac{640\times947}{1528} = 396.65$
Science	$\frac{274\times581}{1528} = 104.18$	$\frac{274\times947}{1528} = 169.82$
Engineering	$\frac{614\times581}{1528} = 233.46$	$\frac{614\times947}{1528} = 380.54$

partial χ^2	Female	Male
Lib Arts	$\frac{(378-243.35)^2}{243.35} = 74.50$	$\frac{(262-396.65)^2}{396.65} = 45.71$
Sci	$\frac{(99-104.18)^2}{104.18} = 0.26$	$\frac{(175-169.82)^2}{169.82} = 0.16$
Eng	$\frac{(104-233.46)^2}{233.46} = 71.79$	$\frac{(510-380.54)^2}{380.54} = 44.05$

$$\chi^2 = 74.50 + 45.71 + 0.26 + 0.16 + 71.79 + 44.05 = \boxed{236.47}$$

The $df = (3-1)\times(2-1) = 2 \Rightarrow$ Critical value $\chi^2_{\alpha=.01,df=2} = \boxed{9.21}$

Therefore we **reject** H_0 (at .01 level) and conclude that there is a relationship between gender and major.

R Code & Output

```
table <- matrix(c(378, 99, 104,
                  262, 175, 510), 3, 2)
colnames(table) <- c("Female", "Male")
rownames(table) <- c("Liberal Arts", "Science",
"Engineering")
table
```

```
             Female Male
Liberal Arts    378  262
Science          99  175
Engineering     104  510
```

```
chisq.test(table)
```

```
        Pearson's Chi-squared test

data:  table
X-squared = 236.47, df = 2, p-value <
2.2e-16
```

Take Another Look at the Example

(Proportion)	Female	Male	Total
Liberal Arts	.59 (.65)	.41 (.28)	(.42)
Science	.36 (.17)	.64 (.18)	(.18)
Engineering	.17 (.18)	.83 (.54)	(.40)
Total	.38	.62	1

Rejecting $H_0 \Rightarrow$ conditional probabilities are not consistent with marginal probabilities

Example: Comparing Two Population Proportions

Let $p_1 = \text{P}(Female|Liberal\,Arts)$ and $p_2 = \text{P}(Female|Science)$.

$n_1 = 640, X_1 = 378,\ n_2 = 274, X_2 = 99$

- $H_0 : p_1 - p_2 = 0$ vs. $H_a : p_1 - p_2 \neq 0$
- $z_{obs} = \frac{.59 - .36}{\sqrt{\frac{.52\times.48}{640} + \frac{.52\times.48}{274}}} = 6.36 > z_{0.025} = 1.96$
- We do have enough statistical evidence to conclude that $p_1 \neq p_2$ at .05% significant level.

R Code & Output

```r
prop.test(x = c(378, 99), n = c(640, 274),
          correct = F)
```

```
        2-sample test for equality of
        proportions without continuity
        correction

data:  c(378, 99) out of c(640, 274)
X-squared = 40.432, df = 1, p-value =
2.036e-10
alternative hypothesis: two.sided
95 percent confidence interval:
 0.1608524 0.2977699
sample estimates:
   prop 1    prop 2
0.5906250 0.3613139
```

Example: Test for Homogeneity

Let $p_1 = \mathrm{P}(Liberal\,Arts)$, $p_2 = \mathrm{P}(Science)$, $p_3 = \mathrm{P}(Engineering)$

- The Hypotheses:

  $$H_0 : p_1 = p_2 = p_3 = \frac{1}{3}$$

  H_a : At least one is different

- The Test Statistic:

  $$\chi^2_{obs} = \frac{(640 - 509.33)^2}{509.33} + \frac{(274 - 509.33)^2}{509.33} + \frac{(614 - 509.33)^2}{509.33}$$
  $$= 33.52 + 108.73 + 21.51 = 163.76 > \chi^2_{.05, df=2} = 5.99$$

- Rejecting H_0 at .05 level

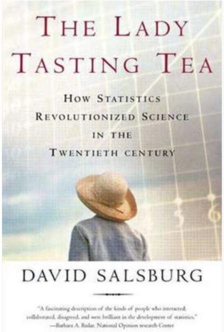
R Code & Output


```
chisq.test(x = c(640, 274, 614), p = rep(1/3, 3))
```

```
        Chi-squared test for given
        probabilities

data:  c(640, 274, 614)
X-squared = 163.76, df = 2, p-value
< 2.2e-16
```

The Lady Tasting Tea



Chapter/Section Title	Page #	Page Count
Author's Preface	vii	
The Lady Tasting Tea	1	8
The Skew Distributions	9	16
That Dear Mr. Gosset	25	8
Raking Over the Muck Heap	33	8
``Studies In Crop Variation"	41	12
``The Hundred-Year Flood"	53	8
Fisher Triumphant	61	12
The Dose That Kills	73	10
The Bell-Shaped Curve	83	10
Testing the Goodness of Fit	93	14
Hypothesis Testing	107	10
The Confidence Trick	117	8

The Lady Tasting Tea Experiment

A lady declares that by tasting a cup of tea made with milk she can discriminate whether the milk or the tea infusion was first added to the cup. We will consider the problem of designing an experiment by means of which this assertion can be tested. [...] [It] consists in mixing eight cups of tea, four in one way and four in the other, and presenting them to the subject for judgment in a random order. The subject has been told in advance of that the test will consist, namely, that she will be asked to taste eight cups, that these shall be four of each kind [...]. — Fisher, 1935.

R Code & Output

```r
TeaTasting <-
matrix(c(3, 1, 1, 3), nrow = 2,
       dimnames = list(Guess = c("Milk", "Tea"),
                       Truth = c("Milk", "Tea")))
TeaTasting
```

```
      Truth
Guess  Milk Tea
  Milk    3   1
  Tea     1   3
```

```r
fisher.test(TeaTasting, alternative = "greater")
```

```
        Fisher's Exact Test for Count Data

data:  TeaTasting
p-value = 0.2429
alternative hypothesis: true odds ratio is greater
than 1
```

Summary

In this lecture, we learned

- Test for Independence
- Test for Homogenelty
- Fisher's Exact Test