STAT 8020 R Lab 5: Multiple Linear Regression I

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Contents

Species diversity on the Galapagos Islands

First Step: Load the data

```
#installinstall.packages("faraway")
library(faraway)
data(gala)

gala
```

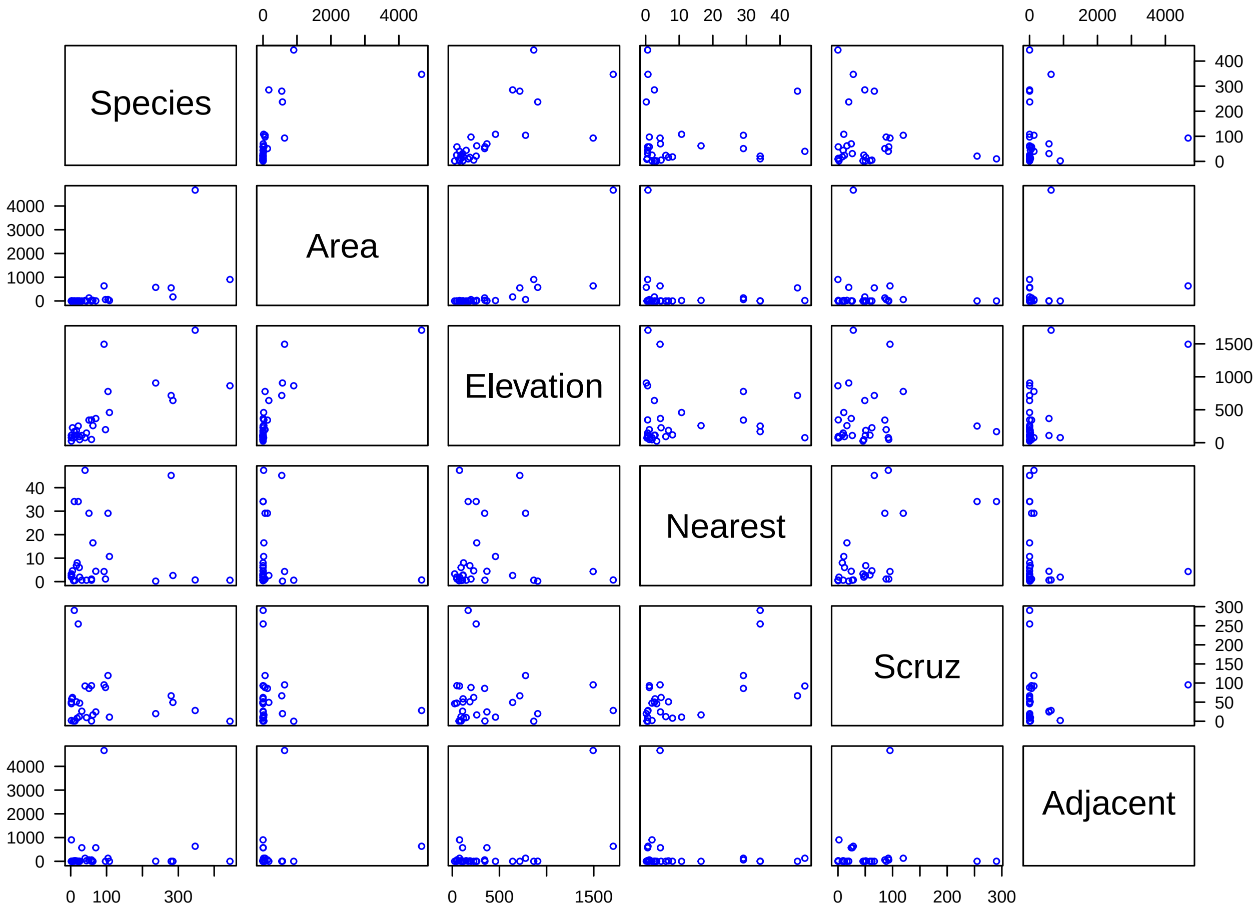
```
##              Species Endemics    Area Elevation Nearest Scruz Adjacent
## Baltra            58       23   25.09       346     0.6   0.6     1.84
## Bartolome         31       21    1.24       109     0.6  26.3   572.33
## Caldwell           3        3    0.21       114     2.8  58.7     0.78
## Champion          25        9    0.10        46     1.9  47.4     0.18
## Coamano            2        1    0.05        77     1.9   1.9   903.82
## Daphne.Major      18       11    0.34       119     8.0   8.0     1.84
## Daphne.Minor      24        0    0.08        93     6.0  12.0     0.34
## Darwin            10        7    2.33       168    34.1 290.2     2.85
## Eden               8        4    0.03        71     0.4   0.4    17.95
## Enderby            2        2    0.18       112     2.6  50.2     0.10
## Espanola          97       26   58.27       198     1.1  88.3     0.57
## Fernandina        93       35  634.49      1494     4.3  95.3  4669.32
## Gardner1          58       17    0.57        49     1.1  93.1    58.27
## Gardner2           5        4    0.78       227     4.6  62.2     0.21
## Genovesa          40       19   17.35        76    47.4  92.2   129.49
## Isabela          347       89 4669.32      1707     0.7  28.1   634.49
## Marchena          51       23  129.49       343    29.1  85.9    59.56
## Onslow             2        2    0.01        25     3.3  45.9     0.10
## Pinta            104       37   59.56       777    29.1 119.6   129.49
```

```
## Pinzon          108     33   17.95     458   10.7  10.7    0.03
## Las.Plazas       12      9    0.23      94    0.5   0.6   25.09
## Rabida           70     30    4.89     367    4.4  24.4  572.33
## SanCristobal    280     65  551.62     716   45.2  66.6    0.57
## SanSalvador     237     81  572.33     906    0.2  19.8    4.89
## SantaCruz       444     95  903.82     864    0.6   0.0    0.52
## SantaFe          62     28   24.08     259   16.5  16.5    0.52
## SantaMaria      285     73  170.92     640    2.6  49.2    0.10
## Seymour          44     16    1.84     147    0.6   9.6   25.09
## Tortuga          16      8    1.24     186    6.8  50.9   17.95
## Wolf             21     12    2.85     253   34.1 254.7    2.33
```

```r
#Out the data in csv
#write.csv(gala, file = "gala.csv")
```

Plot the pairwise scatterplots

```r
plot(gala[, -2], cex = 0.75, col = "blue", las = 1)
```



Correlation matrix

```r
cor(gala[, -2])
```

```
##              Species      Area  Elevation     Nearest       Scruz
## Species   1.00000000 0.6178431 0.73848666 -0.01409407 -0.17114244
```

```
## Area       0.61784307  1.0000000  0.75373492 -0.11110320 -0.10078493
## Elevation  0.73848666  0.7537349  1.00000000 -0.01107698 -0.01543829
## Nearest   -0.01409407 -0.1111032 -0.01107698  1.00000000  0.61541036
## Scruz     -0.17114244 -0.1007849 -0.01543829  0.61541036  1.00000000
## Adjacent   0.02616635  0.1800376  0.53645782 -0.11624788  0.05166066
##              Adjacent
## Species    0.02616635
## Area       0.18003759
## Elevation  0.53645782
## Nearest   -0.11624788
## Scruz      0.05166066
## Adjacent   1.00000000
```

Model 1: Fitting a simple linear regression

Here we use *Elevation* as the predictor as it has the highest correlation with *Species*

```
M1 <- lm(Species ~ Elevation, data = gala)
summary(M1)
```

```
##
## Call:
## lm(formula = Species ~ Elevation, data = gala)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -218.319  -30.721  -14.690    4.634  259.180
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 11.33511   19.20529   0.590     0.56
## Elevation    0.20079    0.03465   5.795 3.18e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 78.66 on 28 degrees of freedom
## Multiple R-squared:  0.5454, Adjusted R-squared:  0.5291
## F-statistic: 33.59 on 1 and 28 DF,  p-value: 3.177e-06
```

Model 2: Adding *Area*

```
M2 <- lm(Species ~ Elevation + Area, data = gala)
summary(M2)
```

```
##
## Call:
## lm(formula = Species ~ Elevation + Area, data = gala)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -192.619  -33.534  -19.199    7.541  261.514
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 17.10519   20.94211   0.817  0.42120
```

```
## Elevation    0.17174    0.05317   3.230  0.00325 **
## Area         0.01880    0.02594   0.725  0.47478
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 79.34 on 27 degrees of freedom
## Multiple R-squared:  0.554,  Adjusted R-squared:  0.521
## F-statistic: 16.77 on 2 and 27 DF,  p-value: 1.843e-05
```

Model 3: Adding *Adjacent*

```r
M3 <- lm(Species ~ Elevation + Area + Adjacent, data = gala)
summary(M3)
```

```
##
## Call:
## lm(formula = Species ~ Elevation + Area + Adjacent, data = gala)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -124.064  -34.283   -8.733   27.972  195.973
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.71893   16.90706  -0.338  0.73789
## Elevation    0.31498    0.05211   6.044  2.2e-06 ***
## Area        -0.02031    0.02181  -0.931  0.36034
## Adjacent    -0.07528    0.01698  -4.434  0.00015 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 61.01 on 26 degrees of freedom
## Multiple R-squared:  0.746,  Adjusted R-squared:  0.7167
## F-statistic: 25.46 on 3 and 26 DF,  p-value: 6.683e-08
```

Full Model

```r
M4 <- lm(Species ~ Elevation + Area + Adjacent + Nearest + Scruz, data = gala)
summary(M4)
```

```
##
## Call:
## lm(formula = Species ~ Elevation + Area + Adjacent + Nearest +
##     Scruz, data = gala)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -111.679  -34.898   -7.862   33.460  182.584
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.068221  19.154198   0.369 0.715351
## Elevation    0.319465   0.053663   5.953 3.82e-06 ***
## Area        -0.023938   0.022422  -1.068 0.296318
```

```
## Adjacent    -0.074805   0.017700  -4.226 0.000297 ***
## Nearest      0.009144   1.054136   0.009 0.993151
## Scruz       -0.240524   0.215402  -1.117 0.275208
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 60.98 on 24 degrees of freedom
## Multiple R-squared:  0.7658, Adjusted R-squared:  0.7171
## F-statistic:  15.7 on 5 and 24 DF,  p-value: 6.838e-07
```

Parameter Estimation

```
X <- model.matrix(M4)
y <- gala$Species
# regression parameters
beta_hat <- solve(t(X) %*% X) %*% t(X) %*% y
#beta_hat_faster <- solve(crossprod(X), crossprod(X, y))
# fitted values
y_hat <- X %*% solve(t(X) %*% X) %*% t(X) %*% y
```

ANOVA

```
anova(M4)
```

```
## Analysis of Variance Table
##
## Response: Species
##           Df Sum Sq Mean Sq F value    Pr(>F)
## Elevation  1 207828  207828 55.8981 1.023e-07 ***
## Area       1   3307    3307  0.8895 0.3550197
## Adjacent   1  73171   73171 19.6804 0.0001742 ***
## Nearest    1   2909    2909  0.7823 0.3852165
## Scruz      1   4636    4636  1.2469 0.2752082
## Residuals 24  89231    3718
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

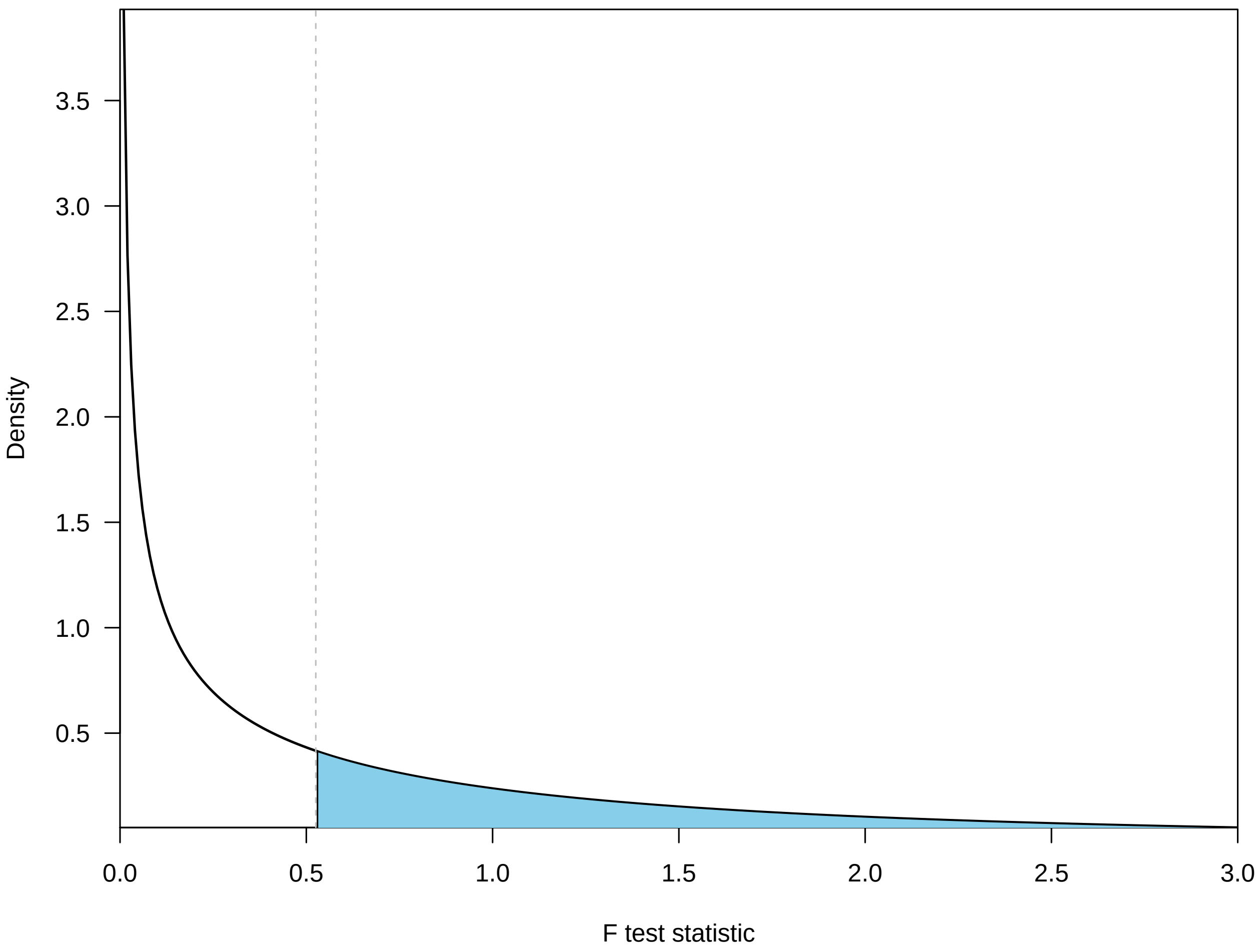
General Linear Test

```
anova(M1, M2)
```

```
## Analysis of Variance Table
##
## Model 1: Species ~ Elevation
## Model 2: Species ~ Elevation + Area
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1     28 173254
## 2     27 169947  1      3307 0.5254 0.4748
```

```
par(las = 1, mar = c(4.1, 4.1, 1.1, 1.1))
xg <- seq(0, 3, 0.01)
yg <- df(xg, 1, 27)
plot(xg, yg, type = "l", xaxs = "i", yaxs = "i", lwd = 1.6,
     xlab = "F test statistic", ylab = "Density")
```

```r
abline(v = 0.5254, lty = 2, col = "gray")
polygon(c(xg[xg > 0.5254], rev(xg[xg > 0.5254])),
        c(yg[xg > 0.5254], rep(0, length(yg[xg > 0.5254]))),
        col = "skyblue")
```

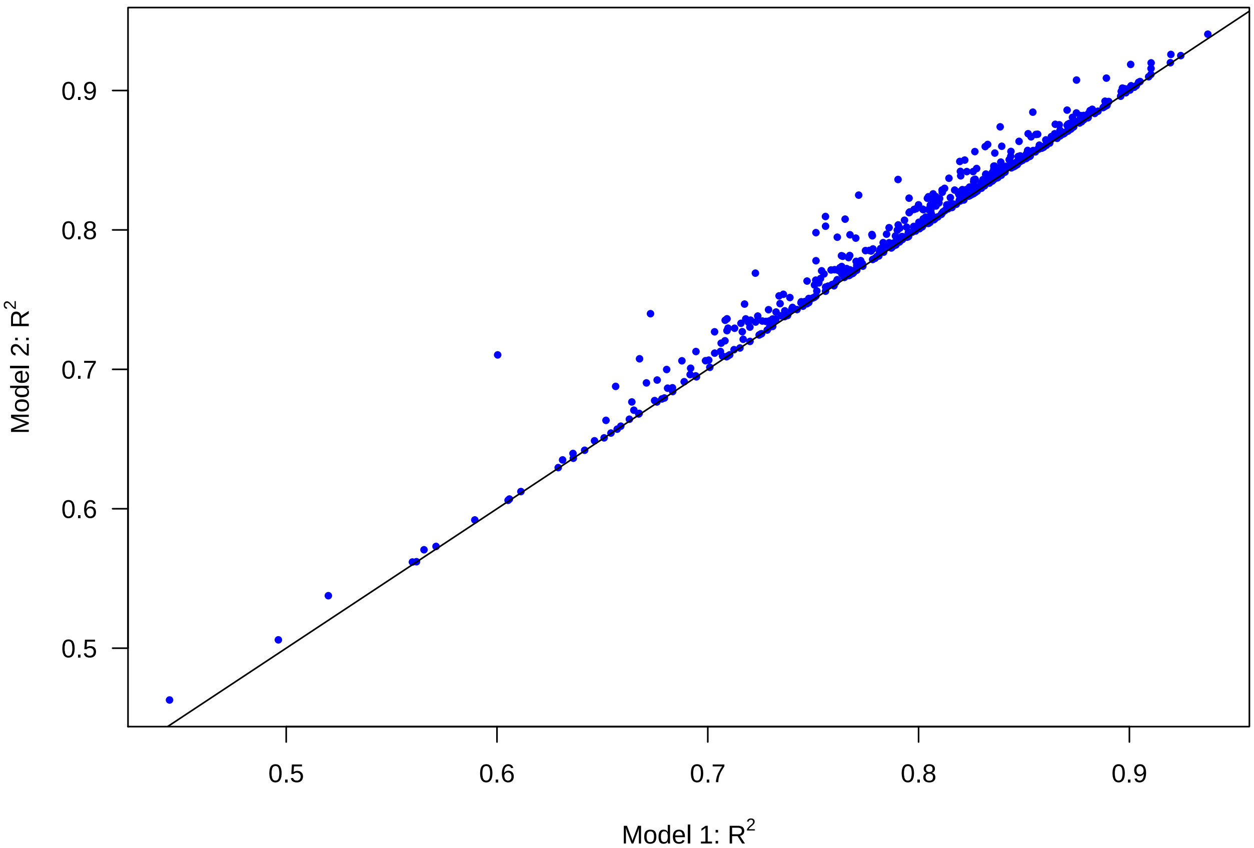


Simulation

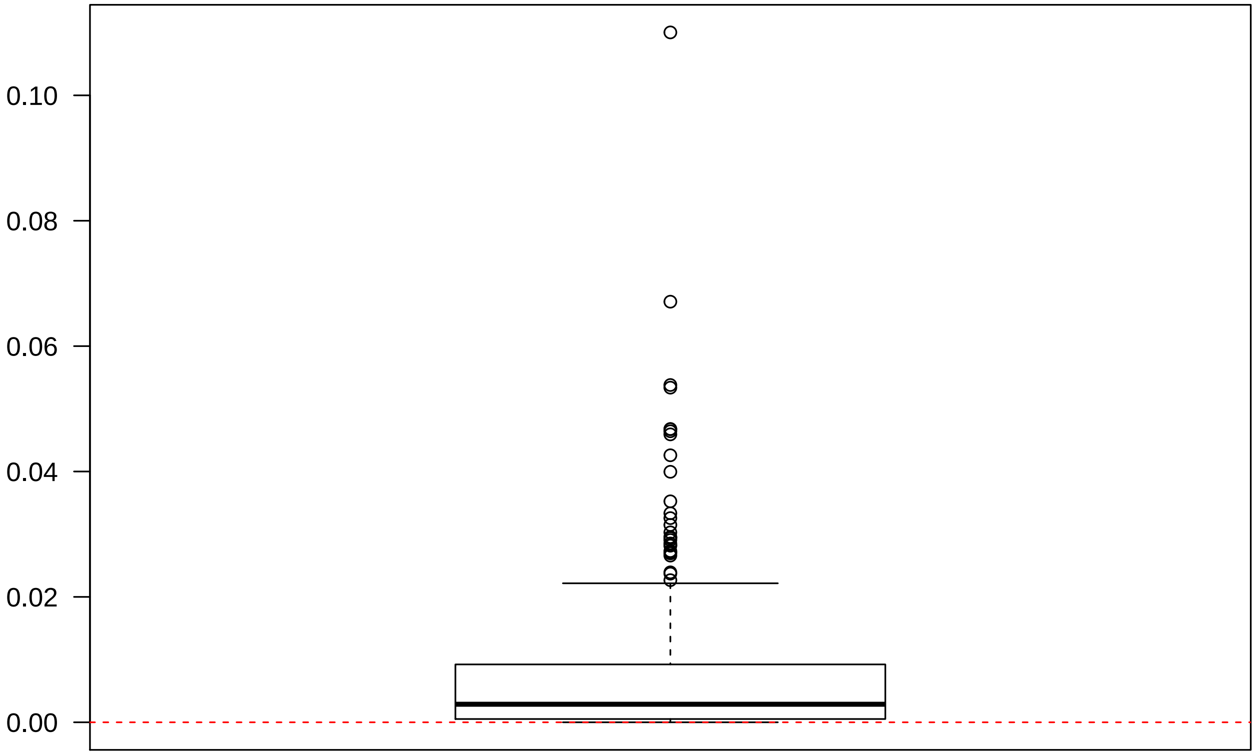
R^2 vs. R^2_{adj}

```r
set.seed(123)
N = 500
x1 <- replicate(N, rnorm(30))
x2 <- replicate(N, rnorm(30))
y1 <- apply(x1, 2, function(x) 5 + 2 * x + rnorm(30, 0, 1))
R.sq <- array(dim = c(N, 4))
for (i in 1:N){
  R.sq[i, 1] = summary(lm(y1[, i] ~ x1[, i]))$r.squared
  R.sq[i, 2] = summary(lm(y1[, i] ~ x1[, i]))$adj.r.squared
  R.sq[i, 3] = summary(lm(y1[, i] ~ x1[, i] + x2[, i]))$r.squared
  R.sq[i, 4] = summary(lm(y1[, i] ~ x1[, i] + x2[, i]))$adj.r.squared
}
```

```r
par(las = 1)
plot(R.sq[, 1], R.sq[, 3], pch = 16, cex = 0.65, col = "blue",
     xlab = expression(paste("Model 1: ", R^2)),
     ylab = expression(paste("Model 2: ", R^2)))
abline(0, 1)
```

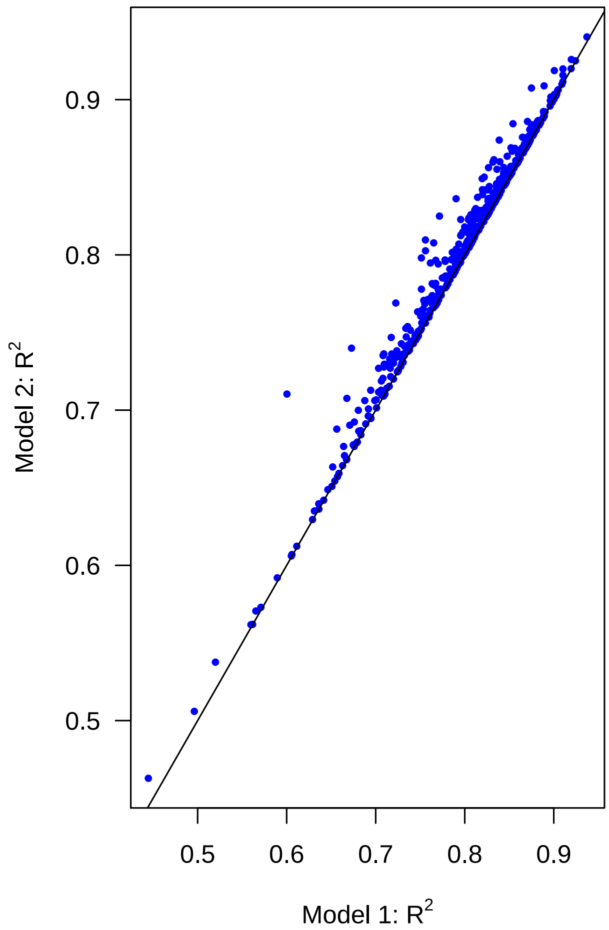


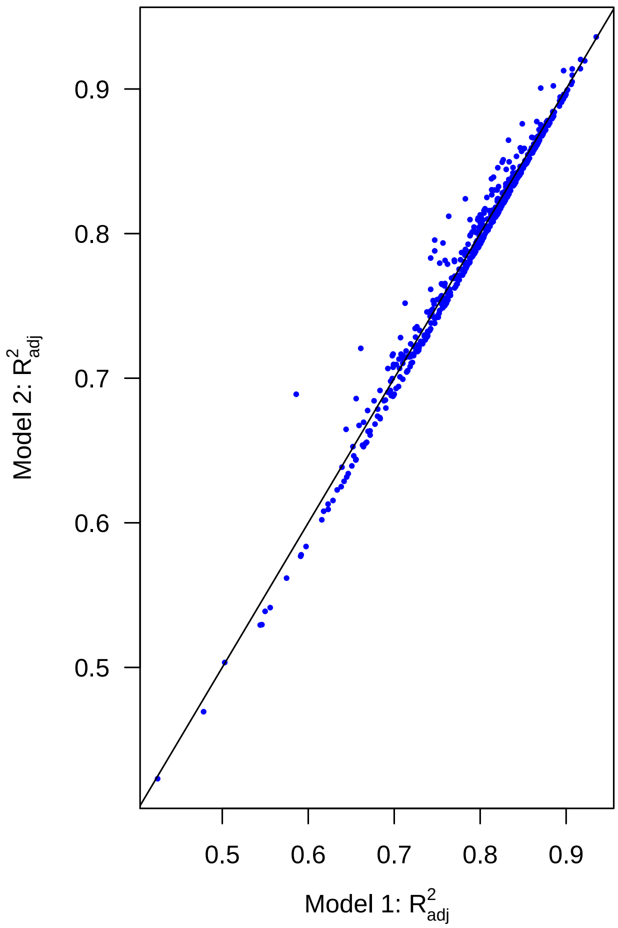

```r
boxplot(R.sq[, 3] - R.sq[, 1], las = 1)
abline(h = 0, lty = 2, col = "red")
```




```r
par(las = 1, mfrow = c(1, 2), mar = c(5.1, 4.6, 1.1, 1.1))
plot(R.sq[, 1], R.sq[, 3], pch = 16, cex = 0.65, col = "blue",
     xlab = expression(paste("Model 1: ", R^2)),
     ylab = expression(paste("Model 2: ", R^2)))
abline(0, 1)

plot(R.sq[, 2], R.sq[, 4], pch = 16, cex = 0.5, col = "blue",
     xlab = expression(paste("Model 1: ", R[adj]^2)),
     ylab = expression(paste("Model 2: ", R[adj]^2)))
abline(0, 1)
```



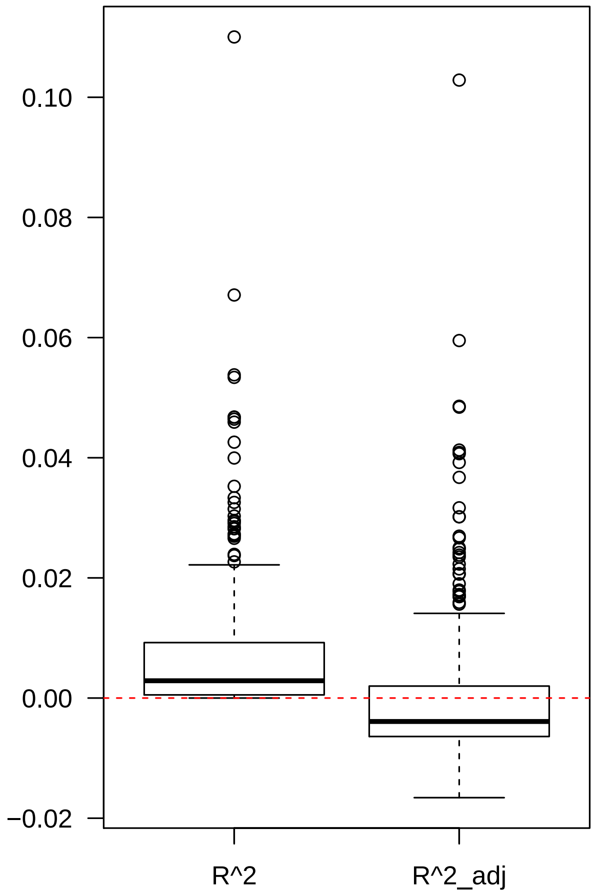


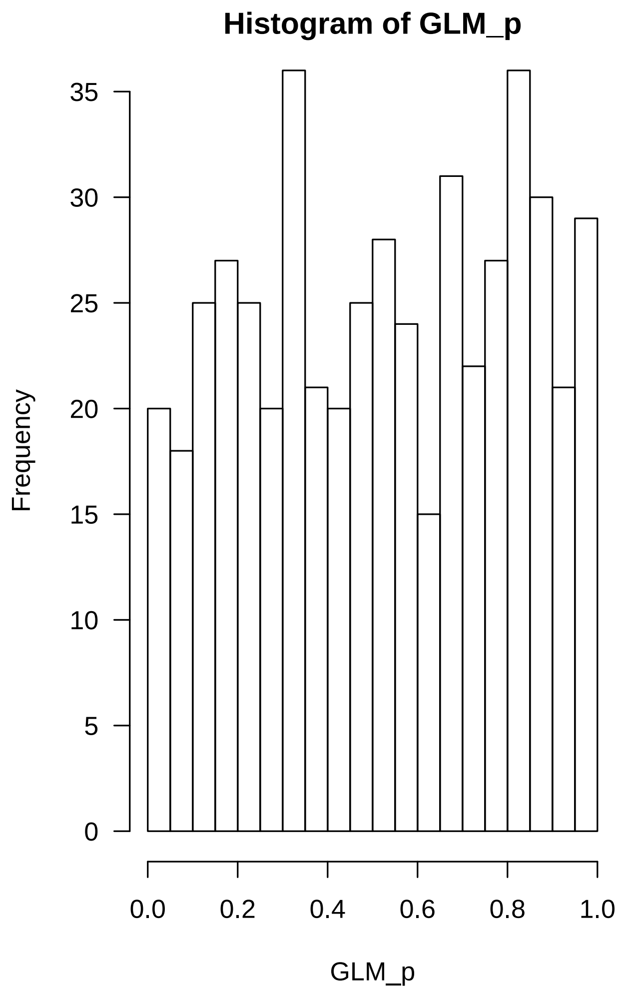

```
boxplot(R.sq[, 3] - R.sq[, 1], R.sq[, 4] - R.sq[, 2], las = 1)
abline(h = 0, lty = 2, col = "red")
axis(1, at = 1:2, labels = c("R^2", "R^2_adj"))

GLM_p <- numeric(500)

for (i in 1:500){
  reduce <- lm(y1[, i] ~ x1[, i])
  full <- lm(y1[, i] ~ x1[, i] + x2[, i])
  GLM_p[i] <- anova(reduce, full)$`Pr(>F)`[2]
}

hist(GLM_p, 30, las = 1)
```





Multicollinearity

```
library(MASS)

x <- replicate(N, mvrnorm(n = 30, c(0, 0), matrix(c(1, 0.9, 0.9, 1), 2)))
y <- array(dim = c(30, N))
for (i in 1:N){
  y[, i] = 4 + 0.8 * x[, 1, i] + 0.6 * x[, 2, i] + rnorm(30)
}
beta <- array(dim = c(3, N))
for (i in 1:N){
  beta[, i] <- lm(y[, i] ~ x[, 1, i] + x[, 2, i])$coefficients
}
```