Lecture 10

Multiple Linear Regression VI

Reading: Chapter 13

STAT 8020 Statistical Methods II

September 22, 2020

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Agenda

1. **Regression with Both Quantitative and Qualitative Predictors**

2. **Polynomial Regression**

Regression with Both Quantitative and Qualitative Predictors

Multiple Linear Regression

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_{p-1} X_{p-1} + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0, \sigma^2)$$

$X_1, X_2, \cdots, X_{p-1}$ are the predictors.

Question: What if some of the predictors are qualitative (categorical) variables?

$\Rightarrow$ We will need to create **dummy (indicator) variables** for those categorical variables

Example: We can encode `Gender` into 1 (Female) and 0 (Male)

Salaries for Professors Data Set

The 2008-09 nine-month academic salary for Assistant Professors, Associate Professors and Professors in a college in the U.S. The data were collected as part of the on-going effort of the college's administration to monitor salary differences between male and female faculty members.

```
> head(Salaries)
       rank discipline yrs.since.phd yrs.service  sex salary
1      Prof          B            19          18 Male 139750
2      Prof          B            20          16 Male 173200
3  AsstProf          B             4           3 Male  79750
4      Prof          B            45          39 Male 115000
5      Prof          B            40          41 Male 141500
6 AssocProf          B             6           6 Male  97000
```

Predictors

```
> summary(Salaries)
       rank     discipline yrs.since.phd    yrs.service   
 AsstProf : 67   A:181      Min.   : 1.00   Min.   : 0.00  
 AssocProf: 64   B:216      1st Qu.:12.00   1st Qu.: 7.00  
 Prof     :266              Median :21.00   Median :16.00  
                            Mean   :22.31   Mean   :17.61  
                            3rd Qu.:32.00   3rd Qu.:27.00  
                            Max.   :56.00   Max.   :60.00  
     sex          salary      
 Female: 39   Min.   : 57800  
 Male  :358   1st Qu.: 91000  
              Median :107300  
              Mean   :113706  
              3rd Qu.:134185  
              Max.   :231545  
```

We have three categorical variables, namely, `rank`, `discipline`, and `sex`.

Dummy Variable

For binary categorical variables:

$$X_{\texttt{sex}} = \begin{cases} 1 & \text{if } \texttt{sex} = \text{male}, \\ 0 & \text{if } \texttt{sex} = \text{female}. \end{cases}$$

$$X_{\texttt{discip}} = \begin{cases} 0 & \text{if } \texttt{discip} = \text{A}, \\ 1 & \text{if } \texttt{discip} = \text{B}. \end{cases}$$

For categorical variable with more than two categories:

$$X_{\texttt{rank1}} = \begin{cases} 0 & \text{if } \texttt{rank} = \text{Assistant Prof}, \\ 1 & \text{if } \texttt{rank} = \text{Associated Prof}. \end{cases}$$

$$X_{\texttt{rank2}} = \begin{cases} 0 & \text{if } \texttt{rank} = \text{Associated Prof}, \\ 1 & \text{if } \texttt{rank} = \text{Full Prof}. \end{cases}$$

Design Matrix

```
> head(X)
  (Intercept) rankAssocProf rankProf disciplineB yrs.since.phd
1           1             0        1           1            19
2           1             0        1           1            20
3           1             0        0           1             4
4           1             0        1           1            45
5           1             0        1           1            40
6           1             1        0           1             6
  yrs.service sexMale
1          18       1
2          16       1
3           3       1
4          39       1
5          41       1
6           6       1
```

With the design matrix $\boldsymbol{X}$, we can now use method of least squares to fit the model $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$

Model Fit:

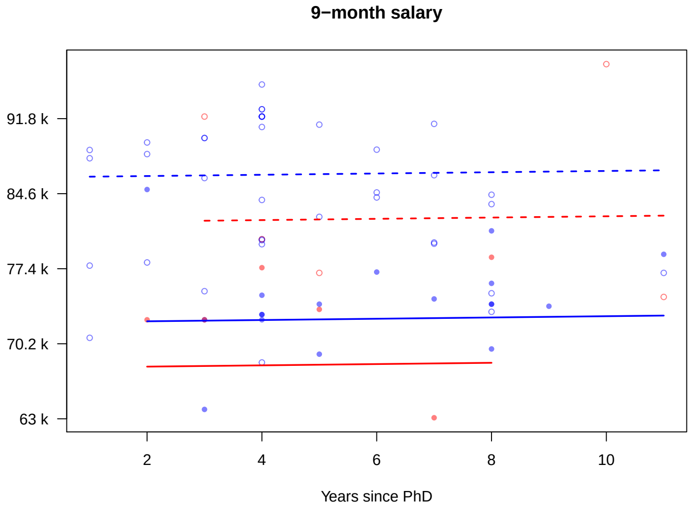
$\texttt{lm}(\texttt{salary} \sim \texttt{rank} + \texttt{sex} + \texttt{discipline} + \texttt{yrs.since.phd})$

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   67884.32    4536.89  14.963  < 2e-16 ***
disciplineB   13937.47    2346.53   5.940 6.32e-09 ***
rankAssocProf 13104.15    4167.31   3.145  0.00179 **
rankProf      46032.55    4240.12  10.856  < 2e-16 ***
sexMale        4349.37    3875.39   1.122  0.26242
yrs.since.phd    61.01     127.01   0.480  0.63124
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 22660 on 391 degrees of freedom
Multiple R-squared:  0.4472,    Adjusted R-squared:  0.4401
F-statistic: 63.27 on 5 and 391 DF,  p-value: < 2.2e-16
```

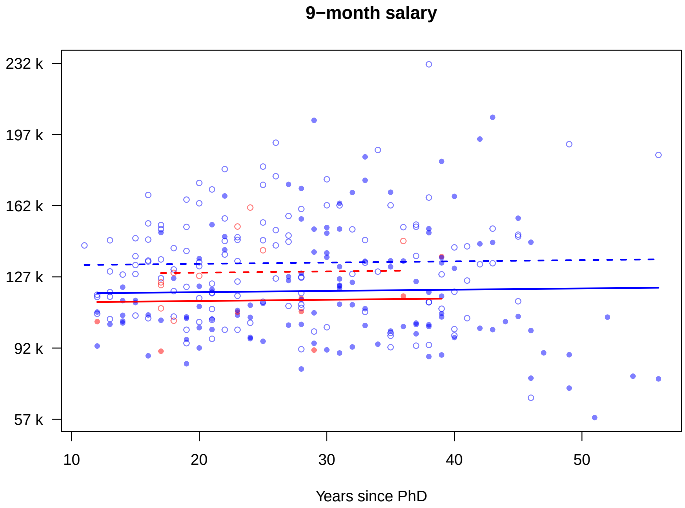
Question: Interpretation of the slopes of these dummy variables (e.g. $\hat{\beta}_{\texttt{rankAssocProf}}$)? Interpretation of the intercept?

Model Fit for Assistant Professors



Model Fit for Associate Professors

Model Fit for Full Professors



lm(salary ∼ sex ∗ yrs.since.phd)

lm(salary ∼ disp * yrs.since.phd)

Polynomial Regression

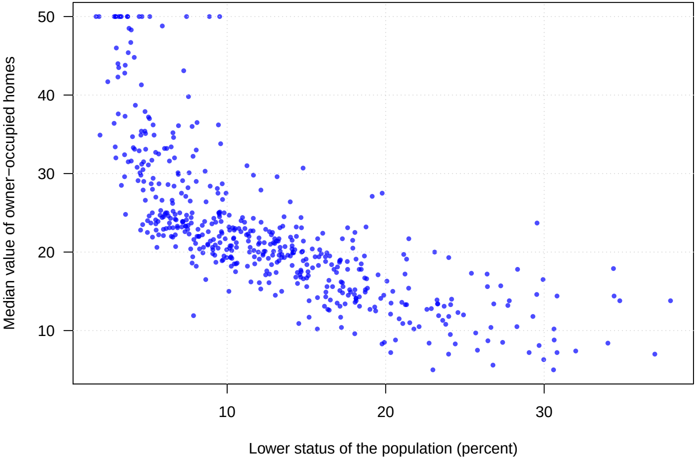
Suppose we would like to model the relationship between response Y and a predictor X as a p_{th} degree polynomial in X:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \cdots + \beta_p X_i^p + \varepsilon$$

We can treat polynomial regression as a special case of multiple linear regression. In specific, the design matrix takes the following form:

$$\boldsymbol{X} = \begin{pmatrix} 1 & X_1 & X_1^2 & \cdots & X_1^p \\ 1 & X_2 & X_2^2 & \cdots & X_2^p \\ \vdots & \dots & \ddots & \vdots & \vdots \\ 1 & X_n & X_n^2 & \cdots & X_n^p \end{pmatrix}$$

Housing Values in Suburbs of Boston Data Set



Polynomial Regression Fits