Lecture 11

Advanced Topics I

STAT 8020 Statistical Methods II

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Agenda

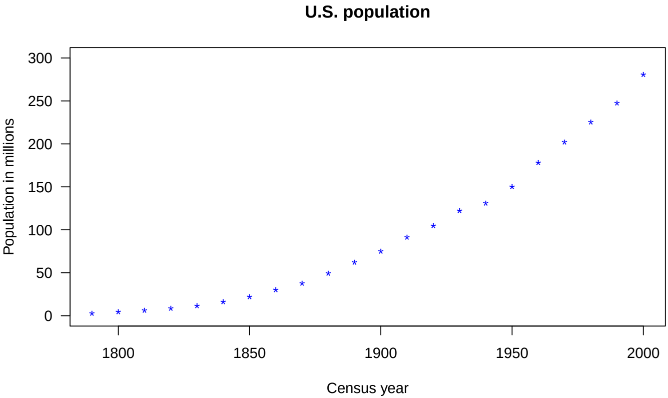
1. **Nonlinear Regression**
2. **Non-parametric Regression**
3. **Ridge Regression**

Moving Away From Linear Regression

- We have mainly focused on linear regression so far
- The class of polynomial regression can be thought as a starting point for relaxing the linear assumption
- In this lecture we are going to discuss non-linear and non-parametric regression modeling

Population of the United States

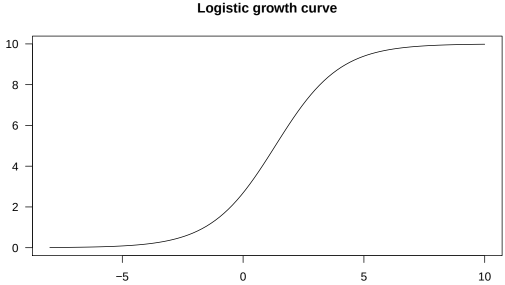
Let's look at the `USPop` data set, a bulit-in data set in R. This is a decennial time-series from 1790 to 2000.



Logistic Growth Curve

A simple model for population growth is the logistic growth model,

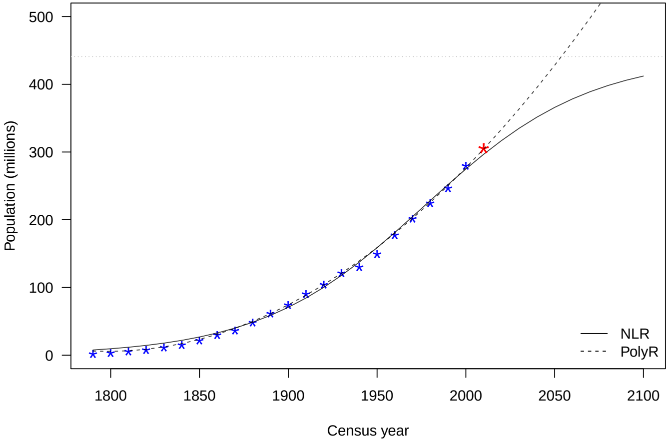
$$
\begin{aligned}
Y &= m(X, \boldsymbol{\phi}) + \varepsilon \\
&= \frac{\phi_1}{1 + \exp\left[-(x - \phi_2)/\phi_3\right]} + \varepsilon
\end{aligned}
$$



We are going to fit a logistic growth curve to the U.S. population data set

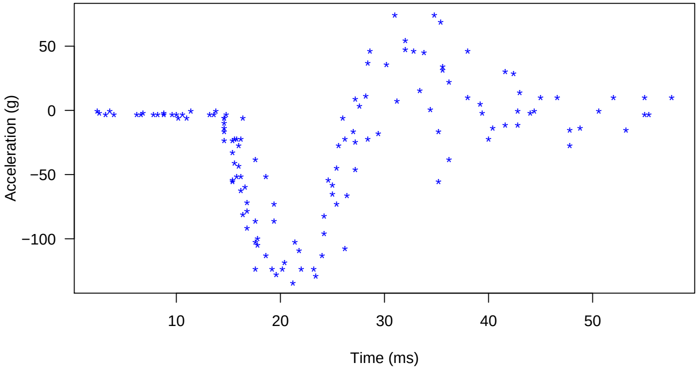
Fitting logistic growth curve to the U.S. population

$$\hat{\phi}_1 = 440.83,\ \hat{\phi}_2 = 1976.63,\ \hat{\phi}_3 = 46.29$$



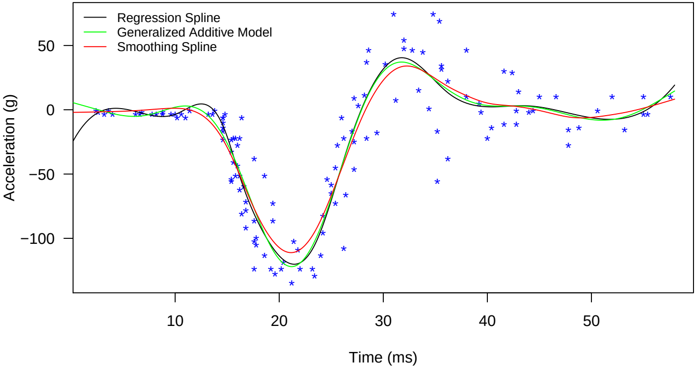
Non-parametric Regression

Let's use the motor-cycle impact data as an illustrative example. This data set is taken from a simulated motor-cycle crash experiment in order to study the efficacy of crash helmets.



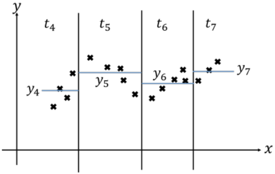
Non-parametric Regression Fits

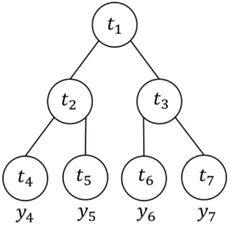
The main idea “non-parametric” regression modeling is to fit the data “locally”. Therefore, no global structure assumption made when fitting the data.



Regression Tree

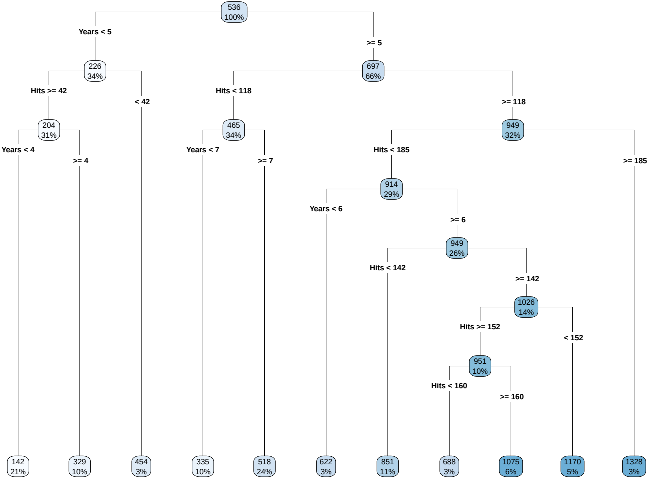
- Partitioning X-space into sub-regions and fit simple model to each sub-region
- The partitioning pattern is encoded in a tree structure





We will use Major League Baseball Hitters Data from the 1986–1987 season to give you a quick idea of what a regression tree might look like

Regression Tree



Longley's Economic Regression Data

We are going to use Longley's data set, which provides a well-known example of multicollinearity, to illustrate Ridge regression.

	GNP.deflator	GNP	Unemployed	Armed.Forces	Population	Year	Employed
1947	83.0	234.289	235.6	159.0	107.608	1947	60.323
1948	88.5	259.426	232.5	145.6	108.632	1948	61.122
1949	88.2	258.054	368.2	161.6	109.773	1949	60.171
1950	89.5	284.599	335.1	165.0	110.929	1950	61.187

Linear Regression Fit

```
Call:
lm(formula = response ~ ., data = trainingData)

Residuals:
   1960    1948    1953    1949    1947    1959    1954    1962    1958
-0.2393  0.9650  0.6495 -0.7423 -0.3187 -0.3387  0.1607 -0.1808  1.2922
   1956    1957    1955
 0.3738  0.3889 -2.0104

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -2.232e+04  2.332e+04  -0.957    0.382
GNP          -1.596e-01  4.535e-01  -0.352    0.739
Unemployed   -8.768e-02  1.138e-01  -0.770    0.476
Armed.Forces -5.346e-02  5.626e-02  -0.950    0.386
Population   -1.331e+00  1.322e+00  -1.007    0.360
Year          1.173e+01  1.210e+01   0.970    0.377
Employed     -3.918e+00  3.498e+00  -1.120    0.314

Residual standard error: 1.284 on 5 degrees of freedom
Multiple R-squared:  0.9939, Adjusted R-squared:  0.9866
F-statistic: 136.2 on 6 and 5 DF,  p-value: 2.251e-05
```

The Predictor Variables are Highly Correlated

	GNP	Unemployed	Armed.Forces	Population	Year	Employed
GNP	1.00	0.60	0.45	0.99	1.00	0.98
Unemployed	0.60	1.00	-0.18	0.69	0.67	0.50
Armed.Forces	0.45	-0.18	1.00	0.36	0.42	0.46
Population	0.99	0.69	0.36	1.00	0.99	0.96
Year	1.00	0.67	0.42	0.99	1.00	0.97
Employed	0.98	0.50	0.46	0.96	0.97	1.00

GNP	Unemployed	Armed.Forces	Population	Year	Employed
14350.70398	601.69137	98.18754	558.11084	22897.44840	1064.78369

Ridge Regression as Multicollinearity Remedy

- Recall least squares suffers because $(\boldsymbol{X}^T\boldsymbol{X})$ is almost singular thereby resulting in highly unstable parameter estimates

- Modification of least squares that overcomes multicollinearity problem

$$\hat{\boldsymbol{\beta}}_{\text{ridge}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \left(\tilde{\boldsymbol{Y}} - \boldsymbol{Z}\boldsymbol{\beta}\right)^T \left(\tilde{\boldsymbol{Y}} - \boldsymbol{Z}\boldsymbol{\beta}\right) \quad \text{s.t.} \sum_{j=1}^{p-1} \beta_j^2 \leq t,$$

  where $\boldsymbol{Z}$ is assumed to be standardized and $\tilde{\boldsymbol{Y}}$ is assumed to be centered

- Ridge regression results in (slightly) biased but more stable estimates and better prediction performance

Ridge Regression Fit

```
Call:
linearRidge(formula = response ~ ., data = trainingData)


Coefficients:
              Estimate Scaled estimate Std. Error (scaled)
(Intercept)  -1.337e+03              NA                  NA
GNP           2.997e-02       1.016e+01           1.973e+00
Unemployed    1.614e-02       4.465e+00           2.033e+00
Armed.Forces  8.106e-03       1.833e+00           1.835e+00
Population    4.732e-02       1.086e+00           4.174e+00
Year          6.940e-01       1.114e+01           1.356e+00
Employed      8.821e-01       1.056e+01           3.988e+00
             t value (scaled) Pr(>|t|)
(Intercept)                NA       NA
GNP                     5.151 2.60e-07 ***
Unemployed              2.196  0.02807 *
Armed.Forces            0.999  0.31800
Population              0.260  0.79480
Year                    8.215 2.22e-16 ***
Employed                2.648  0.00809 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Ridge parameter: 0.01640472, chosen automatically, computed using 2 PCs

Degrees of freedom: model 3.474 , variance 3.104 , residual 3.844
```